An Introduction to Strings and Some of its Phenomenological Aspects [1]

Gerardo Aldazabal
Instituto Balseiro, Bariloche
Argentina

[1] Preliminary notes for internal use

Contents

Chapter 1

Introduction

In these notes we present a brief introduction to string theory. As a motivation we start by exposing some of the many striking achievements of the Standard model of fundamental interactions but also some of its limitations. We introduce string theory through a discussion of perturbative closed bosonic string. Since no knowledge of supersymmetry is assumed the different, consistent, superstrings are just qualitatively motivated. D-branes and open string are then discussed and some rough idea about dualities and non perturbative effects is advanced. We end by illustrating the idea of *brane worlds* as possible framework to embed the Standard model in a string theory context.

Chapter 2

The Standard Model of fundamental interactions and Beyond

The present understanding of the fundamental components of matter and their interactions is based on two very different theoretical descriptions, the Standard model and general relativity. Both descriptions have produced impressive results. However, at some point, they appear as mutually inconsistent.

The fruitful interplay of theory and experiment, over several decades, lead to what is known today as the Standard Model of fundamental interactions. Namely, a theory which is able to describe, in a very successful way, the components of matter and their strong and electroweak interactions. A list of key historical achievements that guided physicist to such a model should contain, among many others:

- 1930: Dirac equation, Rudiments of QED (P. Dirac) β decays, the neutrino proposal (W.Pauli, E.Fermi 1933). Discovery of *muon* in cosmic rays (1937)
- 1940: Quantum electrodynamics. Photon mediated interactions (Feynman, Schwinger, Tomonaga)....
- 1950: Do W bosons mediate weak interactions?
- 1960: Quarks (M. Gell Mann.)
  Standard Model is proposed (S. Glashow, A.Salam, S. Weinberg (1967).
- 1980: Ws y Z discovered at CERN. The number of families must be 3.
- 1990: Top quark discovered at Fermilab (1995). τ-neutrino identified at DONUT (2000)

.

The Standard Model is a relativistic quantum gauge field theory where

Interactions are mediated by gauge (Lorentz) vector bosons associated to the gauge group $G_{\rm SM} = SU(3) \times SU(2) \times U(1)_Y$ while

Matter spectrum of particles includes three fermionic families (or generations) of quarks and leptons.

All such particles have been detected in experiments.

A last, peculiar, scalar particle, the Higgs, is needed in the full construction of the Standard model in order to break electroweak symmetry down to electromagnetism $SU(2) \times U(1)_Y \to U(1)_{em}$.

		SU(3)	SU(2)	$U(1)_Y$	$U(1)_{\mathrm{em}}$
quarks	$Q_L^f = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix}$	3	2	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
	u_R^f	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$
	d_R^f	$\bar{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$
leptons	$E_L^f = \begin{pmatrix} \nu_L^f \\ e_L^f \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	e_R^f	1	1	1	1
	ν_R^f	1	1	0	0
Higgs	$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
gauge bosons	G	8	1	0	0
	W	1	3	0	$(0, \pm 1)$
	B	1	1	0	0

Table 2.1: The particle content of the Standard Model. The index $f = 1, 2, 3$ labels the three families of chiral quarks Q_L^f, u_R^f, d_R^f and chiral leptons l_L^f, e_R^f. All of them are Weyl fermions and transform in the $(\frac{1}{2}, 0)$ representation of the Lorentz group The subscripts R, L do not specify the representation of the Lorentz group but instead are used to indicate the different transformation properties under the chiral gauge group $SU(2) \times U(1)$. The electromagnetic charge is defined by $Q_{em} = T^3_{SU(2)} + Q_Y$.

The Higgs boson had not been detected yet (see [1]). The content of the Standard Model is presented in Table 1.

Let us emphasize that fermions listed in the table are left -handed $(\frac{1}{2}, 0)$ Weyl fermions. We see that the fermion content of the model is *chiral*. Namely, there are no left handed Weyl fermions with conjugate quantum numbers. For instance there is a $(\bar{3}, 1)_{1/3}$ (u_R) but not a $(3, 1)_{-1/3}$ particle. Would such a state exist then this particle would be non chiral, a so called vector-like particle (Of course, for each of the above particles we have the corresponding right-handed $((0, \frac{1}{2}))$Weyl fermion antiparticle with conjugate quantum numbers).

Chirality is a fundamental property of the SM. It is at the origin of parity violation. It also forbids the presence of fermionic (Dirac) mass terms in the Lagrangian. Fermions acquire masses through couplings to the Higgs particle, the so called Yukawa couplings, in the process of electroweak symmetry breaking. That is through couplings of the form

$$Q_L d_R h; \qquad Q_L u_R h^* \qquad E_L e_R h \tag{2.1}$$

Schematically, a scalar potential requires the neutral component of the Higgs field doublet to acquire a vacuum expectation value $< h >$. Agreement with experimental data indicates that

$$M_W \propto < h > \propto \frac{m_h}{\sqrt{\lambda}} \simeq 10^2 GeV \tag{2.2}$$

and thus, fermion masses are proportional to the electroweak symmetry breaking scale.

There is increasing evidence, from solar, atmospheric and laboratory experiments, of the existence of nonzero neutrino masses (see Roulet's talk in this conference). Thus, a new right handed neutrino ν_R $((1,1,0))$ could be added to the SM spectrum. Notice that, a Majorana mass term, $M\nu_R\nu_R$ is allowed for such a particle.

The Standard Model is a theoretically consistent relativistic quantum model. It is a renormalizable, anomaly free unitary model. Experimental results can be understood from the model with great accuracy and many theroetical predictions have been by experiments. Among the most striking ones we could mention the prediction of the very existence of new particles needed for internal theoretical consistency of the model (like the top quark, for instance). An example of the robustness of the Standard Model is provided by the impressive coincidence between the experimental measurement and the theoretical prediction of the electron gyromagnetic moment up to twelve orders of magnitude. Namely,

$$\begin{aligned} \frac{1}{2} g_e^{exp} &= 1.001159652187 \pm 4 \\ \frac{1}{2} g_e^{theor.} &= 1.001159652140 \pm 28 \end{aligned} \tag{2.3}$$

One of the best agreements in the whole physics. Moreover, it is important to stress that the theoretical value is obtained by considering radiative contributions, [5, 6] in the computation and, therefore, it provides a strong check of the QFT idea itself.

Despite the enormous success of *Quantum* field theory in describing strong and electroweak interactions, it is the *classical general relativity* theory that we use nowadays to describe gravitational interactions. In this context, gravitational interactions are encoded in a classical space-time metric $G\mu\nu$ and an action functional is generated by invoking a general diffeomorfism invariance of physics. For instance, the action for the metric itself reads

$$S_{gravity} = \frac{1}{16\pi G_{Newt}} \int_{X_4} \sqrt{-G}(R + \Lambda) \tag{2.4}$$

where G is the determinant of 4×4 metric tensor, R the corresponding scalar curvature and Λ a possible cosmological constant.

$$G_{Newt} = 6.67310^{-11} m^3 kg^{-1} s^{-2} \tag{2.5}$$

is Newton's gravitational constant.

Classical Einstein theory has been checked from comological scales down to fraction of millimeter lengths ($\simeq 10^{-7}$ m). This is an impressive range of many orders of magnitude. Nevertheless, at short distances, where quantum effects for ordinary (SM) interactions become relevant (nuclear $\sim 10^{-13} m$ or even atomic $\sim 10^{-8} m$) Newton law has not been tested.

In fact, were we to attempt to quantize the theory then, an estimate of the gravitational strength for a particle of mass M, for instance a proton ($M \simeq 1GeV$), is

$$\frac{G_{Newt} M^2}{4\pi \hbar c} \simeq 10^{-40} \tag{2.6}$$

This is an extremely small number compared to the strength of usual SM interactions like, for instance $\alpha_{em} \simeq 1/137$, the electromagnetic fine structure coupling constant.

Quantum gravitational effects, comparable with SM interactions, should manifest at very high energies, namely, for M values such that the left hand side in the above equation is of order one. This is called the Planck mass M_P scale and it is given by

$$M_P = \sqrt{\frac{\hbar c}{G_{Newton}}} \simeq 1.210^{19} GeV \tag{2.7}$$

The smallness of Newton's coupling constant is the reason why we are able to describe our world in a very satisfactory way by using, both, a classical description for gravity and a quantum field theory for the other interactions. Even if we had a quantum theory for gravity, at the energies available in experiments, quantum effects would be negligible.

However, a complete consistent description requires a quantum theory of gravity. The present viewpoint is that general relativity is an effective field theory, the way in which a *quantum theory of gravity* manifests at low energies (at most up to M_P).

The problem is that we do not know how this *quantum theory of gravity* looks like. A naive quantization of the classical theory, by following the usual procedure of field theory does not work. For example by considering a Feynman diagrams expansion, it is found that a growing number of infinities, that cannot be absorbed in redefinitions of the metric or the cosmological constant, do appear. A correspondingly increasing number of counterterms must be included in the action and, hence, such a theory is non-renormalizable. In some sense the situation is similar to Fermi interactions epoch where precise computations in the effective Fermi theory were available even knowing that completion to a full consistent theory was required.

The search for a quantum theory of gravity, valid at all energy scales, whose low energy limit is general Einstein relativity, is one of the most active and creative branches in high energy physics. String theory is a strong candidate.

2.1 Standard Model, Gravity and unification

There are several reasons of different degree of "necessity" that lead to the belief that the Standard Model of particle physics (plus classical general relativity) is an effective QFT that must be extended to a more fundamental theory [7]. We distinguish:

1. Theoretical reasons
2. Experimental reasons
3. "Ideological" reasons

1. Theoretical: A quantum theory of gravity must exist. From the discussion above we know that QFT is not enough and that new ingredients are required. We will deal, in what follows, with String theory, the strongest candidate for Quantum Gravity.
2. Experimental: Although the SM is extremely successful in explaining most experimental facts, there are several observations that point to new physics. Some experiments are actually compelling in this sense.

   Clearly, neutrino masses and mixings must be included, as established from atmospheric, solar and reactor experiments. [4].

   Compelling evidence for the existence of Dark matter (with a non baryonic component) and Dark energy is accumulating from cosmology experiments.

   Cosmic baryon asymmetry cannot be accounted from SM.

Certainly this does not mean that radically new ideas (like supersymmetry, grand unification, extradimensions, extended objects) must be used to solve these empirical problems. If no attention is paid to aspects like naturalness, hierarchy problem or other "ideological-aesthetical " issues, as mentioned in third point, minimal extensions of SM, by adding some few degrees of freedom and interactions, would be enough to account for these experimental facts (see for instance [8].

3. "Ideological": It is a matter of fact that the Standard Model describes nature, at least up to the M_Z scale (and that QG must manifest at Planck scale) but, is there any underlying logic for its structure?.

   Again, the *why?'s* about the structure of the SM have different degrees of "compellness " and they are somehow biased by the knowledge of possible answers to some of them. Let us briefly mention some of the main questions

   - Hierarchy problem: It basically refers to the instability of the mass of Higgs particle, and therefore of the electroweak scale (see eq. 2.2) under radiative corrections. Loop corrections suggest that Higgs mass should be of the order of the biggest scale in the theory, M_P, for instance. Extreme fine tuning is needed to keep it of the order of M_W. *Supersymmetry* solves the hierarchy problem by introducing higgsino, the fermionic partner of the Higgs, whose mass is protected by chiral symmetry. Large extra dimensions could avoid such problem by lowering the scale of gravity etc.
   - Is there a complete (or partial) unification of all interactions at some energy scale. Extrapolation of coupling constants (combined with Susy) point to such a unification scale, at least for SM gauge interactions.
   - **Naturalness problems:**
     There are several quantities in the Standard Model that we would expect to be of order one, that, however, must have very small values, in order to account for experimental results. In this sense the SM is not natural. Hierarchy problem above can be included in this category. Another example is: *The strong CP problem.*
     Whereas a CP violating term
     $$\theta_{QCD} G_{\mu\nu} \tilde{G}^{\mu\nu} \tag{2.8}$$
     can be included in QCD Lagrangian ($G^{\mu\nu}$ is the $SU(3)_c$ gluon field strength and $\tilde{G}$ its dual) with an arbitrary QCD theta parameter, experiments (neutron electric dipole moment) constrains $\theta_{QCD} \leq 10^{-10}$. Several proposal have been presented to solve this problem. One of the most famous ones is the Peccei-Quinn solution which postulates the existence of a new pseudoscalar field, the *axion.*
     The cosmological constant problem:
     Once gravity enters the play then we must deal with one of the toughest puzzles in fundamental physics, the cosmological constant problem [9]. The cosmological constant Λ is essentially the vacuum energy. From 2.4 we see that it is a quantity with mass to the fourth power M^4 dimensions. Thus, if the larger scale in nature is the Planck scale then $\Lambda \propto M^4_{Planck}$. However, experimental bounds require $\Lambda \leq 10^{-120} M^4_{Planck}$ and, moreover, there are recent claims that a value of the order
     $$\Lambda \simeq 10^{-47} GeV^4 \tag{2.9}$$
     is required observationally, which is many orders of magnitude smaller than the expected value.
   - Why are there 3 generations?
   - Is there any rationality relating the 20 free parameters of the SM?
   - Is there an explanation for mass hierarchies of generations?

•

and many other questions. Such kind of questions underly behind the many proposals for extensions of the Standard Model. Supersymmetry, Grand Unified Theories, Extra dimensions, supergravity, . . . , string theory. Each of them, give plausible explanations to some of the items above. Generically, we should say, new questions are opened and some old ones are rephrased.

In these notes we will deal with string theory. Certainly an extension of the SM but different from other approaches in the sense that it is a theory that addresses the most fundamental issue. Namely, *string theory* is thought to provide a consistent quantum mechanical theory of gauge and gravitational interactions.

String theory provides a consistent, order by order (in perturbation theory) finite, ultraviolet completion of general relativity. Einstein theory is obtained as a low energy limit (below a typical M_s string scale).

Moreover, string theory includes gauge interactions and, in certain scenarios to be discussed below, it provides four dimensional gauge interactions and chiral fermions. It imposes constraints to model building. Many ingredients of other proposed extensions of SM are incorporated, now in a well defined consistent theory, which can lead at low energy, to models very close to the Standard Model.

String theory provides a framework to address some fundamental issues like: the black hole information paradox, the origin of chirality, the number of fermion generations, etc.

Needless to say that string theory is not a complete closed theory. Several aspects of it, like the non perturbative behaviour, are only partially known. Only some corners of the full string/M theory are known.

Also, other new questions are now open.

Even if models, close to the SM, can be obtained at low energies other, completely different possibilities very far from the real world, seem also possible, therefore rising doubts about its predictivity power, etc. Some questions like the cosmological constant have no answer, yet, in the context of string theory.

The big question of the big bang origin of the universe is still open, etc.

Chapter 3

A brief introduction to Strings

String theory is a theory under construction. There exists no closed description of the theory at the *strong coupling*. However, some non-perturbative relevant information about the structure of the theory is already available. In particular it is known that string theory contains not only strings but other extended objects. The full theory is referred today as $M/string$ theory. The so called, perturbative string theories, are understood as possible, corner, manifestations of this complete theory.

Still, this perturbative sector is very well known and its study can shed light on the structure of the full theory.

Let us discuss some very qualitative aspects of this perturbative sector (see for instance [10–13].

In QFT particles are identified with point like fundamental objects. String theory proposes the existence of one-dimensional objects, strings. The typical size of such objects is

$$L_S = 1/M_S \tag{3.1}$$

such that, at energy scales well below the string scale $E << M_S$, such one dimensional objects are, *effectively* seen as point-like particles. Namely, the low energy limit (compared to M_S) of string theory should reduce to ordinary QFT.

In some string theory scenarios the string scale is linked to four dimensional Planck scale and $M_S \simeq 10^{18} GeV$, very much bigger that the $1TeV$ of elementary particles experiments. In terms of lengths (recalling that $\bar{h}c \simeq 197 MeV \times fermi$) we see that a resolution of $L_S = \frac{\bar{h}c}{M_S} \propto 10^{-32} cm$ would be needed to see the string structure.

Just like a particle evolving in time describes a world line trajectory, a string sweeps out a two dimensional surface Σ, *the world sheet*. A point on the *world sheet* is parameterized by two coordinates, a "time" t, like in the particle case, and a a "spatial" coordinate σ which parameterizes the extended dimension at fixed t. Thus, a classical configuration of a string in a d-dimensional, Minkowski, space time M_d will be given by the functions

$$X^\mu(t, \sigma) \qquad \mu = 0, \ldots, D-1 \tag{3.2}$$

which are the string coordinates in M_d of a (t, σ) point in Σ.

The classical action for string configuration in M_d, with metric $\eta_{\mu\nu}$, can be written as

$$S_{Polyakov} = -\frac{T}{2} \int_\Sigma d\sigma d\tau \sqrt{-g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \tag{3.3}$$

where a metric $g_{\alpha\beta}$ on the two dimensional surface Σ has been introduced. At the classical level such action describes the area A of the surface Σ spanned by the string.

It is not the aim of this brief introduction to go to a detailed analysis of this action. Let us stress, nevertheless, that it corresponds to a two dimensional field theory coupled to 2-d gravity. The Lorentz content appears here as an internal symmetry. The constant T is the string tension and fixes the string scale,

$$\pi T = M_S^2 = \frac{1}{2\alpha'} \tag{3.4}$$

where the string constant $2\alpha' = L_S^2$ is usually introduced.

The action possesses several global and local symmetries that we will not discuss here. A fundamental one is conformal invariance, namely the invariance of the action under phase redefinitions of 2-d metric ($g_{\alpha\beta} \to e^{\phi(t,\sigma)} g^{\alpha\beta}$). In particular, the 2-d metric can be locally gauged away such that the action now reads

$$S = -\frac{T}{2} \int_\Sigma \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \tag{3.5}$$

i.e. a two dimensional free field theory. The equations of motion are

$$\nabla^2 X^\mu(t,\sigma) = (\partial_t^2 - \partial_\sigma^2) X^\mu(t,\sigma) = 0 \tag{3.6}$$

which describe a relativistic string (notice the relative coefficient between time and spatial coordinates is $v/c = 1$). Solutions to such equations are just a superposition of non interacting harmonic oscillators, the zero modes describing the centre of mass motion of the string.

The kind of superposition will depend on the boundary conditions. Closed or open string boundary conditions are possible.

3.1 Closed strings

If spatial parameter $\sigma \in [0, 2\pi]$ then strings are closed if

$$X^\mu(\sigma + 2\pi) = X(^\mu \sigma) \tag{3.7}$$

and the generic solution can be written as a sum of right (R) and left(L) moving contributions

$$X^\mu(t,\sigma) = X_L^\mu(t+\sigma) + X_R^\mu(t-\sigma) \tag{3.8}$$

$$\begin{aligned} X_R(t-\sigma)^\mu &= x_0^\mu + L_s^2 p_R^\mu (t-\sigma) + \frac{i}{2} L_s \sum_{m\neq 0} \frac{a_m^\mu}{\sqrt{m}} e^{-2im(t-\sigma)}, \\ X_L(t+\sigma)^\mu &= \tilde{x}_0^\mu + L_s^2 p_L^\mu (t+\sigma) + \frac{i}{2} L_s \sum_{m\neq 0} \frac{\tilde{a}_m^\mu}{\sqrt{m}} e^{-2im(t+\sigma)}, \end{aligned} \tag{3.9}$$

Namely, $X^\mu(t,\sigma)$ describes a string with center of mass position $x^\mu = x_0^\mu + \tilde{x}_0^\mu$ and momentum $p^\mu = p_R^\mu + p_L^\mu$, respectively.

By imposing the familiar quantization relations for harmonic oscillators, which read

$$[a_m^\mu, a_n^\nu] = \eta^{\mu\nu} \delta_{m,-n} \tag{3.10}$$

(and similarly for left movers) the string can be rather straightforwardly quantized.

Actually constraints are present, associated to 2-d gauge invariance.

We mentioned that, classically, the theory is conformal invariant. However, when it is quantized, a conformal anomaly is generated, i.e. such symmetry is not preserved, generically, at the quantum level.

This is a familiar situation in QFT where, for example, chiral symmetry is broken when the theory is quantized. In such a case we know that chiral symmetry is recovered for very definite content of fields. This is the case of the Standard Model where, for instance, the addition of a hypercharge charged Weyl fermion would make the theory inconsistent.

In a similar way, cancellation of conformal anomaly is also possible if a definite number, 26, of bosonic fields $X^\mu(t,\sigma)$ is considered.

Interestingly enough, the number of fields is here the dimension of space time where the string moves. We see a first example of how a consistency requirement of the world sheet 2-d theory *constrains space time physics*. Extra (more than 4) dimensions are required for the theory to be well defined. At the spectrum level, conformal invariance ensures that norm of states is definite positive.

String excitations just correspond to harmonic oscillators, *left* and *right*, which are obtained by the action of creation operators applied to the vacuum (plus some constraints). The *physical particles*, in d dimensional space, are associated such "harmonic" excitations of the string.

The mass of such states ($p^2 = -M^2$) can be calculated to be

$$M^2 = \frac{2}{\alpha'}[N + \tilde{N} - 2\frac{2-d}{24}] = \frac{4}{\alpha'}[N - \frac{2-d}{24}] \tag{3.11}$$

where $N = \tilde{N}$, is the Right (Left) oscillator occupation number. We stress that Left and right oscillators are independent besides the "level matching" relation $N = \tilde{N}$ and that

$$M^2 = \frac{4}{\alpha'}(N-1) \tag{3.12}$$

for $d = 26$.

For instance, apart from the vacuum we will have the first excited state

$$\tilde{a}_1^{\mu\dagger} a_1^{\nu\dagger}|0> \tag{3.13}$$

with mass $M^2 = \frac{26-d}{6\alpha'}$

Would such state be massive then it should span a representation of the Lorentz little group $SO(d-1)$. However, it corresponds to a 2-tensor representation (transforming as a product of two vectors) of the Lorentz little group $SO(d-2)$ (like a gauge boson vector spans a vector representations of $SO(d-2)$!). In order for such state to have a correct interpretation we should require it to be massless and therefore $d = 26$. Again, we see that this conformal invariance requirement, ensures a correct interpretation of excitations organized in Lorenz group representations.

The massless 2-tensor can be decomposed into Lorentz irreducible representations. Actually, only 24 transverse degrees of freedom are physical (time and longitudinal degrees of freedom are unphysical, like in the vector boson case). Therefore the decomposition reads

$$\mathbf{24_v} \otimes \mathbf{24_v} = \phi \oplus B_{\mu\nu} \oplus G_{\mu\nu} = \mathbf{1} \oplus \mathbf{276} \oplus \mathbf{299}. \tag{3.14}$$

where $G_{\mu\nu}$ is the symmetric tracelees part, $B_{\mu\nu}$ an antisymmetric tensor (2-form) and Φ is the trace.

For instance

$$G_{\mu\nu} \propto [\frac{1}{2}(\tilde{a}_1^{\mu\dagger} a_1^{\nu\dagger} + \tilde{a}_1^{\mu\dagger} a_1^{\nu\dagger}) - \tilde{a}_1^{\mu\dagger} a_1^{\nu\dagger}]|0> \tag{3.15}$$

this is nothing but the graviton excitation. In the effective, low energy, theory it will generate the graviton field $G_{\mu\nu}(X)$ of $d = 26$ Einstein gravity. We thus see an indication that closed strings can contain a graviton like field!

Besides gravitons, we also notice that the massless spectrum of the theory contains a massless scalar, *the dilaton*, and a two form (antisymmetric) field.

When more, higher order, oscillators are considered, a full infinite tower of massive states is generated. They organize into massive representations of the 26 dimensional Lorentz group. The masses are proportional to M_S and, thus, we will not see such particles at energies much below M_S.

A last comment is in order with respect to the first, non oscillating state, $|0>$. This state is a (Lorentz scalar) tachyon with mass $M^2 = -\frac{4}{\alpha'}$. It signals an instability of the bosonic string (X coordinates are bosonic here). It is not present in superstring theories.

3.1.1 The closed superstring

Superstrings are constructed by adding fermionic degrees of freedom besides the bosonic coordinates X^μ.

Consistency imposes $d = 10$ in such cases. Spinors in $d = 10$ can have two different chiralities, we name them as $\mathbf{8_s}$ and $\mathbf{8_c}$.

A useful way ($SO(8)$ Fock space representation) in which we can visualize them is as state vectors of the form

$$|\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2}> \tag{3.16}$$

where one chirality spinor, lets say $\mathbf{8_s}$ is obtained by choosing an odd number of minus signs while an even number corresponds to $\mathbf{8_c}$. [1] In this representation a vector, which in $d = 10$ dimensions has 8 (transverse) degrees of freedom reads

$$\mathbf{8_v} \equiv |\underline{\pm 1, 0, 0, 0}> \tag{3.17}$$

where underlining means permutations of ± 1 entry.

Different closed consistent superstring theories can be constructed, namely, Type IIB, Type IIA and heterotic $E_8 \times E_8$ and $SO(32)$ superstrings. The different cases arise, essentially, because of the freedom to work with left and right movers independently. We will not discuss the details of such constructions but, just to have a feeling of how they work, let us present the massless content in these cases.

Type IIB closed string

Besides the bosonic degrees of freedom, so called the Neveu-Schwarz (NS) sector, leading to $\mathbf{8_v}$ vector representation in $d = 10$ fermionic spinor partners (so called Ramond sector) $\mathbf{8_s}$ are introduced. This is done in both left and right movers sectors. Thus, the massless spectrum can be organized into :

NS-NS sector

$$\mathbf{8_v} \otimes \mathbf{8_v} = \phi \oplus B_{\mu\nu} \oplus G_{\mu\nu} = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}. \tag{3.18}$$

which is the same kind of structure we discussed in eq.(3.14) but now in $d = 10$ dimensions.

R-R sector

$$\mathbf{8_s} \otimes \mathbf{8_s} = \phi' \oplus B'_{\mu\nu} \oplus D_{\mu\nu\rho\sigma} = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_+. \tag{3.19}$$

which correspond to bosonic (fermion times fermion) $[n]$-forms, $n = 0, 2, 4$

the 4-form is a self dual, $D =^* D$, 8-dimensional ϵ-tensor.

[1] In $d = 4$ dimensions we would have $|\pm\frac{1}{2}>$ for left and right chiral fermions of $SO(2)$

Table 3.1: CONSISTENT STRING THEORIES IN D=10.

		N	bosonic spectrum	
IIA		2	NS-NS	$g_{\mu\nu}$, $b_{\mu\nu}$, ϕ
			R-R	A_μ, $C_{\mu\nu\rho}$
IIB		2	NS-NS	$g_{\mu\nu}$, $b_{\mu\nu}$, ϕ
			R-R	$c^*_{\mu\nu\rho\sigma}$, $b'_{\mu\nu}$, ϕ'
heterotic $E_8 \times E_8$		1	$g_{\mu\nu}$, $b_{\mu\nu}$, ϕ A^a_μ in adjoint of $E_8 \times E_8$	
heterotic $SO(32)$		1	$g_{\mu\nu}$, $b_{\mu\nu}$, ϕ A^a_μ in adjoint of $SO(32)$	
type I $SO(32)$		1	NS-NS	$g_{\mu\nu}$, ϕ
			open string	A^a_μ in adjoint of $SO(32)$
			R-R	$B'_{\mu\nu}$

The NS-NS and R-R spectra together, form the bosonic components of $D = 10$ IIB (chiral) supergravity.

In the NS-R and R-NS sectors we have the products

$$\mathbf{8_v} \otimes \mathbf{8_s} \quad = \quad \mathbf{8_c} \oplus \mathbf{56_s}. \tag{3.20}$$

There are two $\mathbf{56_s}$ identifying the two Type IIB $N = 2$ (16+16 supersymmetry charges) gravitini, with one vector and one spinor index.

Notice that invariance under the exchange of left and right sectors shows up in the closed Type IIB massless spectrum.

Type IIA closed string

Formally, the same steps as in the Type IIB case are followed. However, opposite chirality spinors are considered in left and right sectors. Type IIA theory is non chiral. The masslees spectrum is organized as

$$[\mathbf{8_v} \oplus \mathbf{8_s}] \otimes [\mathbf{8_v} \oplus \mathbf{8_c}] \quad = \quad \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35} \oplus \mathbf{8_s} \oplus \mathbf{8_c} \oplus \mathbf{56_s} + \oplus \mathbf{56_s}. \tag{3.21}$$

Heterotic string

The heterotic string is constructed in a rather peculiar way. It is built by combining the left sector of the bosonic string with the right sector of the Type II superstring!. We learnt that conformal anomaly cancellation in superstring requires $d = 10$ why 26 bosonic fields X are required in the bosonic string. Such apparent inconsistency is solved by taking 10 X^μ bosonic fields with space-time $\mu = 0, \ldots, 9$ indices and 16 X^I bosonic fields where $I = 1, \ldots, 16$ are internal (no space-time indices). Roughly speaking, like states carrying space time indices μ arrange into representations of Lorentz Group $SO(1, 9)$ internal indices now span representations of a gauge group G of rank 16. Again, world sheet consistency conditions limit such groups to be $SO(32)$ or $E_8 \times E_8$.

The massles field content of consistent string theories in $d = 10$ is presented in Table I (including Type I string to be discussed later below)

It is worth noticing that the closed bosonic sector of all these theories contain the graviton and dilaton fields. Except Type I theory, all other string theories also contain a two $NS{-}NS$ antisymmetric form $B_{\mu\nu}$.

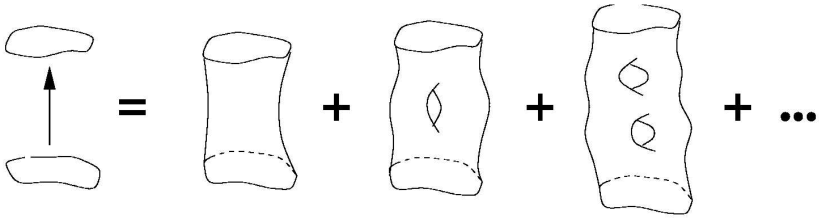

Figure 3.1: Closed string interactions.

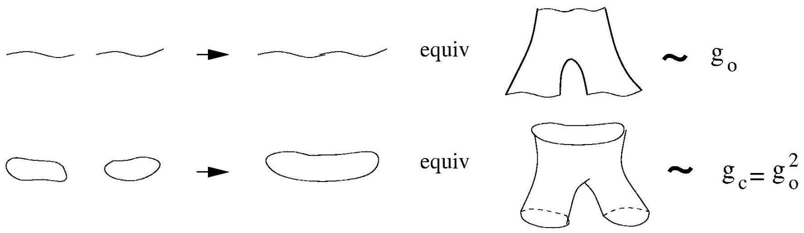

Figure 3.2: Basic interaction vertices for open and closed *(pants-like)* strings

3.2 String interactions

We have identified strings as fundamental objects and its excitations as particles. Still interactions among different strings must be considered. In fact, a well defined prescription for computing scattering among string states can be given. Qualitatively two closed strings evolving in time can join into one closed string and then split again and so o and so forth. Pictorically this is indicated in Fig. 1

We notice that the basic "interaction vertex" is the *short pants*, a Φ^3 like vertex where two strings join into one (or viceversa). Interestingly enough, the interaction is delocalized in space-time in a region of size of the order of L_S. This is the origin of the finiteness of string amplitudes in contrast with QFT where interactions are pointlike.

Formally, the amplitude for a string initial state $|i>$ to evolve to a final configuration $|f>$ is an expression of the form

$$|i> \rightarrow |f> = \sum_{w-sheet \quad topologies} \int [\mathcal{D}X] e^{-S_{Poly}} \mathcal{V}_i(X)\mathcal{V}_f(X) \tag{3.22}$$

Like in QFT a coupling constant g_c is expected to appear associated to each vertex. We see from the figure that the order of a given diagram is related to the number of handles h of the surface spanned by the string evolving in time.

Actually, when we wrote down string action in 3.3 we could have included an extra, topological, term compatible with the symmetries of the theory. Namely, the term

$$S_{euler} = \frac{1}{4\pi\alpha'} \int_\Sigma d\sigma d\tau \sqrt{-g}[\alpha' R^{(2)} \Phi_0] \tag{3.23}$$

where $R^{(2)}$ is the 2d curvature. This term is a total derivative and it is not zero if the surface Σ is not trivial. Actually

$$\chi = \frac{1}{4\pi} \int_\Sigma \sqrt{-g}[\alpha' R^{(2)}] \tag{3.24}$$

is the Euler characteristic of the surface. For a closed surface, it counts the number of handles [2] $\chi = 2(1-h)$. We thus see, that a factor $e^{-\phi_0 \chi}$ will appear associates to a given topology of Euler characteristic χ. If $g_c = e^{\phi_0}$ is small, the perturbative expansion, given in terms of handles, makes sense. The most relevant contribution will be a sphere (g_c^{-2}), then a torus (g_c^0) , a two handle surface (g_c^2), etc.

It is possible to see that ϕ_0 is not a new parameter of the theory but rather can be interpreted as the vacuum expectation value of the dilaton field.

In the limit when the energies under consideration are $E \ll M_S$ these interactions become pointlike and can be described by an effective quantum field theory in $d = 10$ dimensions. This is the point particle limit of string theory.

For instance, for the heterotic string, the (bosonic) part of the effective field theory action (at tree-level (sphere) and up to two-derivative terms) reads

$$S^{\text{het}} = \frac{4}{\alpha'^4} \int d^{10}x \sqrt{G} e^{-2\Phi} \left[R + (\nabla\Phi)^2 - \frac{1}{12}\hat{H}^2 - \frac{\alpha'}{4} F^2 \right] . \qquad (3.25)$$

which is nothing but Einstein general relativity in $d = 10$ dimensions, with metric $G_{\mu\nu}$ coupled to the scalar dilaton field Φ and the antisymmetric form $B_{\mu\nu}$ through the three form $H = dB + ...$ and $F_{\mu\nu}$ is the field strength of the $SO(32)$ or $E_8 \times E_8$ gauge theory. Actually, by a field redefinition

$$\tilde{G}_{\mu\nu} = e^{2\omega} G_{\mu\nu} \qquad (3.26)$$

$$\omega = \frac{1}{4}(\Phi - \Phi_0) \qquad (3.27)$$

the action can be rewritten in a more familiar form in the so called Einstein frame as

$$S_E^{\text{het}} = \frac{1}{2k^2} \int d^{10}x \sqrt{g} \left[R - \frac{1}{8}(\nabla\Phi)^2 + \ldots \right] \qquad (3.28)$$

where $2k^2 = 8\pi G_N = \frac{\alpha'^4}{4} e^{2\Phi_0}$.

Thus, Planck mass and string scales are related as

$$M_P^{(d=10)} = M_S^8 e^{-2\Phi_0} = \frac{M_S^8}{g_c^2} \qquad (3.29)$$

In particular we see that the gravitational coupling constant k depends on the value of ϕ_0. Actually, the shift of the field ϕ by ϕ_0 as above is the correct redefinition to perform if Φ acquires the vacuum expectation valueΦ_0. We thus see that this coupling constant value is not an additional, external parameter of the theory but a value dynamically fixed by the theory itself.

It is worth noticing that a similar observation is valid for other "constants" appearing in string theory. Such constants are actually fixed by the expectation values of the fields of the theory.

3.3 Compactification

All the consistent string theories we have presented are well defined in $d = 10$ dimensional space time. However, string theories in lower, in particular $D = 4$, dimensions can be consistently considered. Actually, when we defined the string world sheet action 3.3 we assumed strings propagating in flat Minkowski space. Namely we chose a space time metric $G_{\mu\nu} = \eta_{\mu\nu}$. However, a more generic situation of strings propagating in curved backgrounds can be studied. Indeed, a general metric tensor $G_{\mu\nu}(X(t, \sigma))$ can be considered.

[2] If b boundaries and c crosscups are present, as it is the case for open strings then $\chi = 2(1-h) - b - c$

Consistency conditions, as conformal invariance on the world sheet, will constrain the possible space time backgrounds. Such allowed backgrounds solutions define the so called *string vacua.* In $d = 10$ dimensions the 4 closed superstring theories plus the open string theory are the only possibilities. In particular, in order to make contact with the physics we know, it is important to look at string theories that, in the low energy limit, lead to effective theories in $d = 4$ space time dimensions.

The key idea is to look at strings propagating in ten dimensional curved X_{10} space-times backgrounds that look like Minkowski four dimensional spacetime times an internal compact 6-dimensional manifold, a so called compactification. Namely

$$X_{10} = M_4 \times X_6 \tag{3.30}$$

Notice that, whenever such no flat situations are allowed, the world sheet action does no longer describe free oscillators but rather becomes an interacting theory. This world sheet action describes a complicated non-linear sigma model that, generically, we are only able to study as an expansion in α'. Thus, in many cases we only know how to deal with the point particle limit.

However, it is possible to consider exact solutions. The simplest cases correspond to toroidal like compactifications. In such cases the internal space is essentially flat Euclidean six dimensional space but where coordinates close up to a torus lattice translation. Namely $X_6 = T^6 \equiv R^6/\Lambda$ where Λ is a six dimensional lattice. As an example consider the simplest case of compactification to $D = 9$ dimensions on a one dimensional torus, namely a circle S_1 of radius R. Then ten dimensional space should look as $X_{10} = M_9 \times S_1$. Thus, let say the ninth string coordinate satisfies

$$X^9(\sigma + 2\pi) = X(^9\sigma) + 2w\pi R \tag{3.31}$$

with w an integer, the *winding number* , instead of 3.7. We see that the solutions to the string theory wave equations are as in the above, flat case, given in Eq.3.9. The difference is in the boundary condition for the ninth coordinate. This condition tells us that this string coordinate can wind w times around the circle of radius R before closing on itself.

Once this condition is imposed we can compute the mass of string excitations $p^\mu p_\mu$ but now with $\mu = 0, 8$. We obtain

$$\frac{\alpha'}{2}M^2 = N + \tilde{N} - 2 + \alpha' \frac{n^2}{4R^2} + \frac{1}{\alpha'}w^2R^2, \qquad N_R - N_L = wn \tag{3.32}$$

This expression contains very interesting features that also manifest in other more involved compactifications.

The last term encodes the fact that for increasing w, the mass of the string excitations must increase since the string is stretched up around the circle. This is a *stringy* effect, not present in field theory.

The second term is just (p^9) momentum quantization as expected from quantization on a box ($e^{ip^9X^9} = e^{ip^9(X^9+2\pi R)}$). By varying n we obtain an infinite tower of massive states with masses $\sim 1/R$; these are the standard 'momentum states' of Kaluza-Klein compactifications in field theory.

In particular, the massless states with $n = m = 0$ and one oscillator in the compact direction

$$a^\dagger{}_\mu \tilde{a}_9^\dagger |0> \tag{3.33}$$

(and the same thing if we exchange left and right) are massless vector fields in the extra dimensions which give rise to a $U(1)_L \otimes U(1)_R$ Kaluza-Klein gauge symmetry. Notice that left and right oscillators, both in compact dimension, give rise to a $9d$ scalar field.

Interestingly enough, for special values of $w \neq 0$ and n extra massless states may appear. In particular for $w = n = \pm 1$ we can see that at the special radius $R^2 = \frac{1}{2}\alpha'$, massless less states with

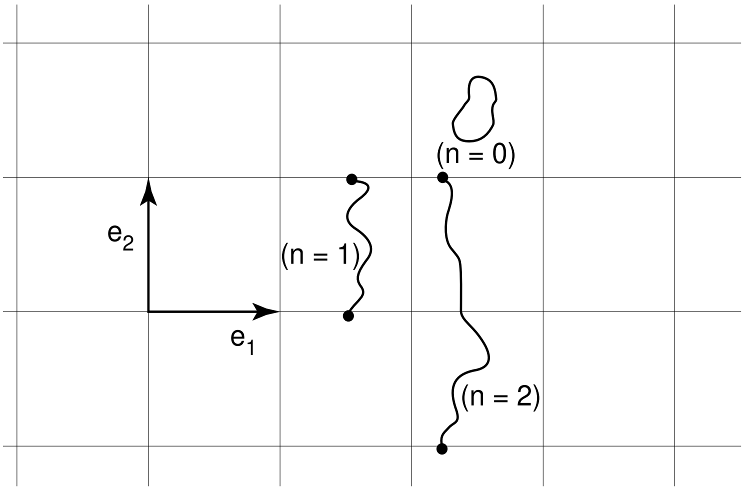

Figure 3.3: A 2D torus T^2 defined by the identification of points on $\mathbb{R}^2$ by elements of the lattice defined by $\mathbf{e_1}$ and $\mathbf{e_2}$. We display examples of a closed string on $\mathbb{R}^2$ which is also closed on T^2 ($n = 0$), also a string closed on T^2 but not on $\mathbb{R}^2$, winding around the torus once($n = 1$) and twice ($n = 2$).

a single oscillator $N = 1, \tilde{N} = 0$ are obtained. Together with the above KK vectors, they contribute to enhance the gauge group to $SU(2)_R \times SU(2)_L$. Again, the radius here is not a new parameter of the theory but rather the expectation value of a dynamical field. If, somehow, the special value $R^2 = \frac{1}{2}\alpha'$ is selected, we refer to it as a point of enhanced symmetry in "moduli space". Since $w \neq 0$, this enhancing is a *stringy* effect.

Finally, let us point out another interesting feature that manifest in the mass expression. Namely, the spectrum is invariant under the exchange

$$R \leftrightarrow \frac{\alpha'}{2R} \qquad w \leftrightarrow n. \tag{3.34}$$

such a transformation is known as T-duality transformation and it is also a stringy property. It exchanges small with large distances and momentum (Kaluza-Klein) states with winding states at the same time. This symmetry can be shown to hold not only for the spectrum but also for the interactions and therefore it is an exact symmetry of string perturbation theory.

The simple case of the circle we have considered can be straightforwardly extended to compactify the six extra dimensions. Strings will be able to wind around the different cycles of the six torus. An example of a T^2 dimensional torus is given in the figure.

In order to make contact with the effective field theory, notice that for low energies and when the volume of the compactification manifold is large compared to string scale,(for instance $\alpha'/R^2 \propto L_S/R \ll 1$)such that no windings or other stringy effects can manifest, we expect the effective (point particle) field theory to be valid and that essentially only the massless modes will be relevant.

As a very simple example consider a $d = 10$ dimensional scalar field $\Phi(x^0, \ldots, x^9)$ on a flat background $X_{10} = M_9 \times S_1$,(this could be for instance the dilaton piece in the heterotic effective action 3.28)

$$S_{10} = \int_{M_9 \times S_1} d^{10}(\nabla\Phi)^2 \tag{3.35}$$

The coordinate $x^9 \in [0, 2\pi R]$ parameterizes the circle here and, therefore, we can expand Φ into its Fourier components

$$\Phi(x^0,\ldots,x^9)=\sum_{nZ}e^{inx^9/R}\Phi^{(n)}(x^\mu) \tag{3.36}$$

with $\mu=0,\ldots,8$. The field equation just reads

$$[P_{9d}^2+P_9^2]\Phi^{(n)}(x^\mu)=0 \tag{3.37}$$

with $P_9=n/R$

Thus, from the $d=9$ dimensional point of view, we have an infinite set of 9d fields $\Phi^{(n)}(x^\mu)$ labelled by the compact 9d momentum n with 9d mass given by

$$M_{9d}^2=(\frac{n}{R})^2 \tag{3.38}$$

These are the Kaluza Klein modes that we identified in 3.32. For energies $E<<M_C\propto 1/R$, much lower than the compactification scale, massive modes are not reachable and only the zero mode $\Phi^{(0)}(x^\mu$ is observable. In such case, the ten dimensional action reduces to an effective $d=9$ dimensional action

$$S_9^{eff}=\int_{M_9}d^9(2\pi R)\Phi^{(0)}\partial_\mu\partial^\mu\Phi^{(0)} \tag{3.39}$$

Notice that, since the x^9 dependence dropped out, the volume of the compact manifold, $V_C=2\pi R$, appears.

We can proceed similarly when fields with non trivial Lorentz transformation are present. The procedure to follow is to decompose the original Lorentz group into the lower dimensional one and then perform the Kaluza Klein reduction. For instance, if we deal with $d=10$ dimensional metric tensor, $G_{MN}(x^0,\ldots,x^9)$ we obtain

$$G_{\mu\nu}(x^0,\ldots,x^9) \rightarrow G_{\mu\nu}^{(0)}(x^\mu) \tag{3.40}$$

$$G_{\mu 9}(x^0,\ldots,x^9) \rightarrow G_{\mu 9}^{(0)}(x^\mu) \tag{3.41}$$

$$G_{99}(x^0,\ldots,x^9) \rightarrow G_{99}^{(0)}(x^\mu) \tag{3.42}$$

these massless modes correspond to the $D=9$ dimensional metric, the $U(1)$ vector field and the scalar we found in 3.32

3.3.1 Scales

By properly choosing the compact manifold X_6 sensible semirealistic theories, close to the Standard Model in the low energy limit, can be obtained when starting with the heterotic string. We briefly refer to them in last chapter. Here we just want to comment about mass scales.

In fact, it is interesting to notice, by following the same steps as above, that the Planck and string scales must be close to each other in such kind of compactifications. Namely, by compactifying the heterotic string action on X_6, with volume V_6, from (3.28) we obtain obtain

$$S_{d=4}^{eff.}\propto\int d^4x\frac{M_S^8V_6}{g_s^2}\left[R_4+\ldots\right]+\frac{M_S^6V_6}{g_s^2}F^2\,. \tag{3.43}$$

Thus, we can express the Planck mass and gauge coupling constant in $d = 4$ dimensions in terms of the string scale and the coupling constant

$$M_P = \frac{M_S^8 V_6}{g_s^2} \simeq 10^{19} GeV \tag{3.44}$$

$$g_{YM} = \frac{M_S^6 V_6}{g_s} \simeq 0.1 \tag{3.45}$$

therefore

$$M_S = M_P g_{YM} \simeq 10^{18} GeV \tag{3.46}$$

Chapter 4

Open strings and D-branes

The world sheet action in (3.3) admits, besides the closed string boundary condition (3.7), open string boundary conditions. In fact, variation of Polyakov action leads to

$$\delta S_{Polyakov} = -\frac{T}{2}\int_{-\infty}^{\infty} dt(g^{\alpha\beta}\delta X^{\mu}\partial_{\beta}X_{\mu})|_{\sigma=0}^{\sigma=l} + \frac{T}{2}\int_{\Sigma} d^2\zeta\delta X^{\mu}\partial_{\alpha}(g^{\alpha\beta}\partial_{\beta}X_{\mu}) \tag{4.1}$$

where the second term leads to the string equations which have a mode expansion solution as in eq. (3.8). In order for the first term to vanish, boundary conditions must be satisfied.

In particular we notice that there are two types of boundary conditions that lead to open strings [10–14]. Namely, Neumann boundary conditions

$$\partial_{\sigma}X = 0 \tag{4.2}$$

at $\sigma = 0, \pi$ or Dirichlet boundary conditions

$$\delta X = 0 \tag{4.3}$$

at $\sigma = 0, \pi$

Let us assume that the coordinates X^{μ} ($\mu = 0, \ldots, p$) satisfy Neumann conditions on both ends, so called NN (Neumann-Neumann) boundary conditions, whereas, coordinates X^I ($I = p+1, \ldots, 9$) satisfy DD boundary conditions. Imposing such conditions in mode expansions we obtain

$$X^{\mu}(t,\sigma) \quad = \quad x' + \; 2\alpha' pt + i\sqrt{2\alpha'}\sum_{m\neq 0}\frac{a_m^{\mu}}{\sqrt{m}}cos(m\sigma)e^{-i\pi t} \tag{4.4}$$

and

$$X^{I}(t,\sigma) \quad = \quad x' + \frac{\delta X'}{\pi}\sigma + \sqrt{2\alpha'}\sum_{m\neq 0}\frac{a_m^{\mu}}{\sqrt{m}}sin(m\sigma)e^{-i\pi t} \tag{4.5}$$

We see that the the end points of X^I string coordinates are fixed. Namely

$$\begin{aligned} X^I(\sigma = 0) &= x_a^I & (4.6)\\ X^I(\sigma = \pi) &= x_a^I + \delta x_I = x_b^I & (4.7)\\ && (4.8) \end{aligned}$$

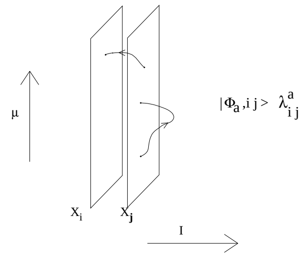

Figure 4.1: D-branes and open strings

while the string is free to move in the other directions.

Thus, the picture that comes out is that of $p+1$ dimensional hyperplanes, so called Dp-branes (D for Dirichlet) where string end points are constrained to live (see figure 4.1). One Dp brane sits at x_a^I while the other at x_b^I. X^I are the coordinates transverse to the Dp-brane [1].

We see that no momentum is allowed to propagate in transverse dimensions and, therefore, the corresponding particles, associated to string excitations, can only propagate along the $p+1$ dimensional "world-volume" of the brane. Notice that for $p = 25$ ($p = 9$ in the susy case) all boundary conditions are NN and the endpoints of the string are free to move in $d = 26$ ($d = 9$) spacetime. However, it is still worth talking about a D25(9)-brane in this case.

The quantization proceeds similarly to the closed string case but now just one kind of oscillators do appear (in fact, boundary conditions require here $a = \tilde{a}$). Also, a string state will be characterized, not only by its world sheet degrees of freedom, i.e. oscillator number, but also by two extra indices a, b , known as Chan -Paton indices, denoting the brane in which the end points of the string are. Namely, an open string state will be characterized by a state vector of the form

$$|\Psi, ab\rangle \tag{4.9}$$

In principle we could have an arbitrary number N of Dp-branes and , thus, $a, b = 1, \ldots, N$.

The mass operator of an open string state can be computed to be

$$M^2 = \frac{1}{\alpha'}(N-1) + \sum_{I=p+1}^{25} \left(\frac{x_b^I - x_a^I}{2\pi\alpha'} \right)^2 \tag{4.10}$$

The last term tells us that stretching the string between two branes requires an energy proportional to brane separation, as expected.

Massless states thus correspond to $N = 1$ oscillators and $x_b^I = x_a^I$ for all (or some of the) possible N values of a, b.

Assume that all Dp- branes coincide on top of each other, then, massless states are

$$\tilde{a}_1^{\mu\dagger}|0, ab\rangle \qquad \mu = 0, \ldots, p \tag{4.11}$$

$$\tilde{a}_1^{I\dagger}|0, ab\rangle \qquad I = p+1 \ldots, 25 \tag{4.12}$$

where the first state describes a gauge vector boson in $p+1$ dimensional space-time. Moreover, since there are N^2 such bosons they will give rise to a unitary $U(N)$ gauge theory in space-time.

Actually a given quantum state vector should be expressable as a combination of the base states $|\psi, ab\rangle$ above. Namely, $|\psi\rangle = \Lambda_{ab}|\psi, ab\rangle$ with Λ_{ab} a hermitian $N \times N$ matrix. Such Chan-Paton matrix spans a representation of $U(N)$.

[1] A more general case is possible with endpoints on Dd and Dq- branes respectively

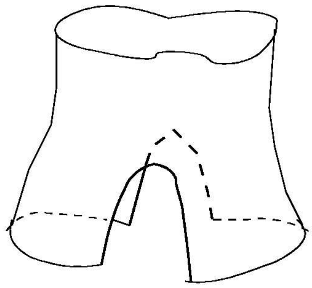

Figure 4.2: Open strings join to create a closed string.

The other massless states, associated to the transverse coordinates, are scalars transforming in the adjoint representation of the gauge group. The presence of transverse oscillations are an indication that the Dp- brane is not just a fixed hyperplane but a dynamical object.

Such $25-p$ scalars can be interpreted as Goldstone bosons associated to the breaking of translational invariance by the Dp-brane.

Suppose that we separate some of the D-branes such that M of the locations of the D-branes in transverse space coincide, then, the associated $|ab\rangle$ states are massless while the rest become massive. The gauge group associated to the stack of coincident branes thus brakes from $U(N)$ down to $U(M)$. In particular notice that, even if all D-branes are separated apart, the $|aa\rangle$ states are massless and the gauge group is thus $U(1)^M$.

We see that moving continuously the locations of the D-branes away from each other leads to the breaking of $U(M) \to U(1)^M$. Such a breaking corresponds to a Higgs breaking generated by diagonal components of scalar field on the D-branes acquiring vev's. Recall that such scalars are associated to transverse coordinates.

As we found in the closed bosonic string a tachyon scalar $N = 0$ appears here. The tachyon is not present when supersymmetry is included [2].

We have introduced the basic ideas about open strings and D branes through the bosonic formulation. Similar steps can be followed in order to introduce a fermionic sector in a *supersymmetric* way (a consistent GSO projection must be performed). Indeed, supersymmetry play a crucial role in the discussion of D-branes which appear as stable BPS states. The gauge theory defined on the world volume is a supersymmetric gauge theory. In particular, a D3-brane, with four dimensional world volume, defines a $d = 4$ $\mathcal{N} = 4$ supersymmetric gauge theory.

We mentioned that presence of open strings necessarily implies the existence of closed ones. Pictorially this is associated to the presence of a "vertex" of the form indicated in Figure 4.2. A world sheet perturbative expansion for an open string evolving from an initial state $|i>$ to a final state $|f>$ will be given in terms of the world sheet surfaces with different topologies.

In particular, consider the 1-loop open string diagram corresponding to the vacuum to vacuum amplitude. The diagram corresponds to an annulus (cylinder) with fixed string endpoints, as shown in fig 4.3 (of course, all such amplitudes, that we describe qualitatively here, can be explicitely computed) at branes A and B.

By exchanging the role of the parameters $\sigma \to \tau$ such diagram can be interpreted as a closed string that propagates in time from one brane to the other, see fig. 4.4.

Namely, the open string one-loop vacuum amplitude can be computed as

$$< B|evolution \quad of \quad closed \quad string|A > \tag{4.13}$$

[2]It is worth mentioning that open tachyons have a clearer interpretation, as brane instabilities, than closed ones [15].

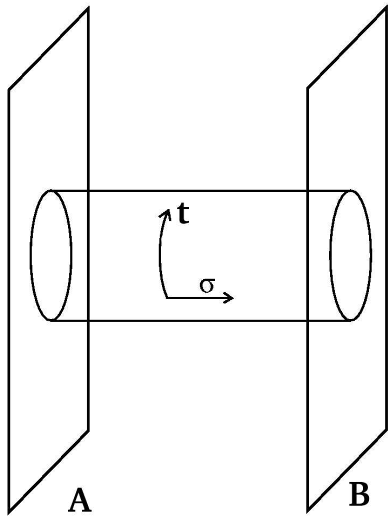

Figure 4.3: One loop open string amplitude.

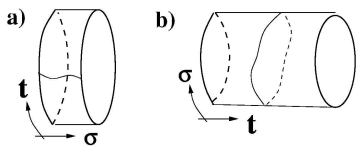

Figure 4.4: Open-closed duality.

from a quantum state $|A>$ to a state $|B>$ [3].

These two ways of interpreting these amplitude are a manifestation of what is known as a *open-closed duality.*

From the closed string point of view, closed string states, that can couple to the Dp-brane, do propagate along the cylinder. In fact, such states can be isolated by stretching the cylinder.Which is the closed string theory from which such closed string states arise? It can be shown that, if supersymmetry is required and the closed-string duality is ensured, such closed string states correspond to the *Type IIB superstring theory* whenever p is *odd*, whereas *Type IIA superstring theory* states are found for even p.

This observation has deep roots. Indeed, it can be shown (we briefly discuss it below) that p-brane extended objects, with p odd (even), do appear as classical non trivial solitonic solutions of the effective field theory associated to type IIB (A) closed superstring theory (TypeIIB(A) sugra). D-branes do provide a microscopic description of solitonic solutions of such sugra theories!.

From the space time point of view we expect a p extended object, the Dp-brane, to be charged with respect to a p-form (antisymmetric p-tensor fields) of the theory. Namely, terms of the type

$$Q_p \int_{W_{p+1}} C_{p+1} \tag{4.14}$$

should appear with Q_p the Dp-brane charge with respect to the C_{p+1} form of the closed sector. This is nothing but a generalization of the coupling of the electromagnetic vector field, a 1-form (A^μ) to the world line of a charged particle, i.e. the electron, which can be thought as a $p = 0$ (0-brane) extended object. Another example is the $B_{\mu\nu}$ 2-form coupling to the $p = 1$ string. C_{p+1} forms are present in Type IIB(A) theory, with p even (odd), they arise from RR sectors (we saw some of them above) that is why we refer to them as RR forms.

In particular, a C_{10} is present in Type IIB which couples to D9-branes as,

$$N_9 \int_{W_{9+1}} C_{10} \tag{4.15}$$

where N_9 is the number of D9-branes (a D9-brane charge is normalized to 1).

This term needs some explanation since we are talking about a [10]-form potential which we did not find when we studied the massless spectrum.

The [10]-form potential is rather peculiar. In fact, notice that it is not possible to construct a field strength $F = dA$ from this form since it would be an [11]-form in $D = 10$. Therefore a kinetic $\int^* F.F$ term in the action is not allowed and thus C_{10} does not represent propagating particles, even if the above couplings are possible. On the other hand, the ten dimensional coupling above is acceptable from Lorentz invariance.

Interestingly enough, the variation of the action (4.15) would lead to inconsistent equations of motion unless $N_9 = 0$ and, therefore, no branes should be present.

Such an inconsistency can be directly observed from the computation of the closed string amplitude represented in figure 4.3. In fact, it can be interpreted as a tadpole divergence.

The conclusion is that it is not possible to couple Type IIB closed string to an open string sector in a consistent, 10d, Poincare invariant way.

There are ways to overcome this difficulty.

In fact, if $d = 10$ Poincare invariance is not required then consistent models can be constructed. Tadpole cancellation equations, associated to lower dimensional [p]-form RR charge cancellation, will still impose restrictions on brane configurations.

[3]such quantum states associated to D-branes can be given a precise mathematical description. They are called boundary states.

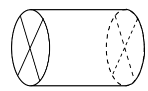

Figure 4.5: Klein-bottle amplitude.

Anti-Dp branes, which are similar to Dp-branes but carrying opposite RR-charge can also be included. Notice that charge cancellation would require now $N_9 = \bar{N}_9$ if $\bar{N}_9$ is the number of antibranes. However, unless some obstruction appears, this is an unstable situation and branes and antibranes do annihilate.

A way out to this situation is to consider unoriented strings. They give rise to the consistent Type I open string theory (which also contains a closed sector). Its massless spectrum is summarized in Table I. We present a very brief description below.

4.1 Type I open theory

Let us introduce a formal operator Ω that, when acting on open strings, exchanges string orientation. In terms of world sheet parameters it corresponds to $\sigma \to -\sigma$.

The open string end points transform as $(a, b) \to (b, a)$. An unoriented string theory is generated if strings related by Ω are identified.

Such unoriented open strings do not couple to Type IIB but to a truncation of it obtained by implementing an Ω projection on the closed sector. Such projection is referred as *orientifolding* the theory.

Actually, Ω orientifold action on closed sector exchanges Left and Right moving sectors. In terms of world sheet parameters it corresponds to $t + \sigma \to t - \sigma$. Namely, two states of the form

$$|\alpha >_L |\beta >_R \qquad |\beta >_L |\alpha >_R \tag{4.16}$$

present in Type IIB theory (recall that Type IIB theory as well as closed bosonic theory is invariant under $L \leftrightarrow R$) are equivalent in the projected theory.

For bosonic string oscillators, for instance,

$$\Omega\colon \quad \alpha^\mu_m \leftrightarrow \tilde{\alpha}^\mu_m. \tag{4.17}$$

and therefore, projecting onto invariant states (achieved by introducing the projector $\frac{1}{2}(1+\Omega)$) we see, for instance that from the original massless spectrum in 3.14, Graviton and dilaton field are kept but the antisymmetric tensor is projected out. The same considerations are valid for Type IIB. However, in the RR sectors, exchanging Left and Right introduces an extra minus sign since R states are fermionic. Thus, for instance, the antisymmetric form is kept in the RR sector (see Table I).

The computation of closed string amplitudes over L-R invariant states can be achieved by just introducing the projector $\frac{1}{2}(1+\Omega)$ in the Type IIB amplitudes. In particular, for the torus vacuum amplitude, while the first term produces just the original torus amplitude (times a half factor) the second term introduces a completely different topology, the Klein bottle. This diagram is shown in figure 4.5. It describes the evolution of an initial left-right state that glues back to itself up to the action of Ω. Actually, as suggested in the figure, it can be also interpreted as a closed string propagating between planes, orientifold O_9 planes, which are left invariant under the orientifold action.

The Klein bottle closed string amplitude is ill defined. It contains tadpole like divergences. It is just these divergences that can cancel out against D9-brane tadpole ones to render the full theory (closed plus open sectors) consistent. In fact such tadpoles can be associated to RR charges of the orientifold O_9 planes. An explicit calculation gives $Q_{O_9} = -32$ and therefore tadpoles are absent if

$$N_9 = 32 \tag{4.18}$$

D9 branes are introduced in the theory!

A stack of N_9 D9 branes would give rise to a $U(N_9)$ unitary gauge theory in the case of oriented strings. However, since open strings with reversed end points must be identified, a subgroup, orthogonal $SO(32)$ subgroup of the original unitary group is obtained.

If we represent the action of the group operator Ω as a unitary matrix γ_Ω, then, the original open string states encoded in Chan- Paton Λ factors ($N_9 \times N_9$ hermitian matrices) which are left invariant by the orientifold projection must satisfy

$$\Lambda = -\gamma_\Omega \Lambda^T \gamma_\Omega^{-1} \tag{4.19}$$

Since $\Lambda = 1$ can be chosen (from tadpole cancellation conditions) , then

$$\Lambda = -\Lambda^T \tag{4.20}$$

which tells us that the gauge group is $SO(32)$.

Chapter 5

Beyond perturbation theory: p-branes and duality

We have found that Dp-branes are extended objects whose fluctuations are described by open string excitations. Such extended objects interact with closed string states. In particular a Dp-brane couples to the graviton (such coupling defining the brane tension) and it carries charge with respect to RR forms.

Interestingly enough such objects do appear as non-trivial, solitonic solutions, of closed string theory effective actions. Namely, by considering the effective low energy actions corresponding to different closed string theories, it is possible to show that finite energy classical solutions to the equations of motion exist. Such solutions correspond to objects which look like lumps of localized energy in some, lets say $d-p-1$ dimensional space, while they extend in the other p spatial dimensions. They are called p-branes. Their energy per unit volume (tension) is proportional to the inverse of the coupling constant and, therefore, these branes are intrinsically non-perturbative (some references for these subjects are [10, 12, 16, 17]). As an example, a 3-brane solution in Type IIB theory reads

$$
\begin{aligned}
ds^2 &= f(r)^{-1/2}[(dx_0)^2 + \ldots (dx_3)^2] + f(r)^{1/2}[(dx_4)^2 + \ldots (dx_9)^2] && (5.1)\\
f(r) &= 1 + 4\pi g_s {\alpha'}^2 N \frac{1}{r^2} && (5.2)
\end{aligned}
$$

with $r^2 = (x_4)^2 + \ldots + (x_9)^2$ (an F_5 form solution is also present) (N is the brane charge). Such a solution is depicted in fig. (5.1) These solutions have many relevant and striking properties.

On the one hand for each effective closed string theory action there exist a p-brane solution for values of p that correspond to the $p+1$ forms present in the (perturbative) closed spectrum of the string theory.

Moreover, p-branes carry electric charge under $p+1$ forms and they are magnetically charged

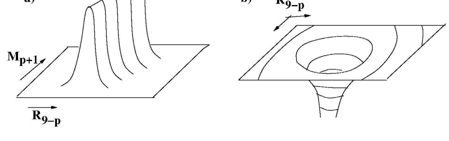

Figure 5.1: D3-brane solution.

Theory	p-brane content	forms
Type IIB	F1,NS5	$B_2, \tilde{B}_6$
	D(-1),D1,D3,D5,D7	$a, \tilde{B}_2, C_4, \tilde{C}_6, \tilde{C}_8$
Type IIA	F1,NS5	$B_2, \tilde{B}_6$
	D0,D2,D4,D6,D8	$C_1, C_3, \tilde{C}_5, \tilde{C}_7$
Heterotic	F1,NS5	$B_2, \tilde{B}_6$
Type I	D1, D5	$B_2, \tilde{C}_6$

Table 5.1: Some p-branes for different superstring theories

under the dual $(7-p)$ forms.

The 3-brane above is an example since we know that a 4-form field is present in Type IIB. The 3-brane is identified in this case with the D3-brane that we have found above, that couples to Type IIB closed states. A list of such solutions is given in Table 5 for each closed string effective action. As mentioned, the energy per unit volume of these solitonic objects is of the order of $\frac{M_s}{g_s}$ or $\frac{M_s}{g_s{}^2}$. For weak coupling $g_s \ll 1$ they are non perturbative and extremely massive.

A crucial result, that we can just sketch here, is that these solutions are 1/2 BPS states. Roughly speaking that means that these solutions are kept invariant by half of the total supersymmetry generators of the theory. Like in any group theory representation, representations of supersymmetry are constructed by keeping together all states that mix up under the symmetry group. Since in this case only half of the susy generators are effective, the corresponding multiplets are shorter than a generic susy multiplet, in fact, they contain half of the number of states.

Why are 1/2 BPS states so important? The fact is that we have studied solutions of effective theories which are actually found in the low energy $\alpha' \to 0$ limit. The action is written in terms of the light modes of the corresponding string theory. Therefore it appears controversial to interpret solutions of this action, describing a particular regime, as solutions of the full string theory. Interestingly enough, if we continuously increase the value of α' we cannot expect a discrete jump from a BPS multiplet to a state which contains twice the degrees of freedom. Namely, such states remain BPS even if α' is turned on. They are stable. [1] The idea is that if a BPS solution of the effective supergravity action is found, with its corresponding mass, charge, etc. there will exist a corresponding BPS state with the same properties in the full string theory. It is said that BPS states are protected by supersymmetry.

5.1 p-brane democracy

Several results point towards the conclusion that different p-branes should be considered on equal footing. Namely, even if some of such branes could appear as more fundamental in some regimes and others as solitonic objects their role could be inverted. For instance, 1-branes, "strings" appear as solitonic solutions. Moreover, in some cases they couple (electrically) to a NS-NS 2-form with the same charge the usual string couples to it. For instance, in Table 5 above the solitonic 1-brane of the heterotic effective action must be interpreted as the *fundamental* heterotic string. However, the fact that we see it as fundamental, in the sense that we can write a world sheet action and quantize it etc., is interpreted as due to the fact that we are in a regime where the coupling constant is small and thus allows us to perform such an expansion in string oscillation modes. If we move to a different regime, lets say of strong coupling, such a perturbative expansion will not be possible and such string will not look as fundamental any more. Other objects could look as more fundamental in this other regime.

[1] Moreover, it can be shown that the mass (tension) of the BPS state coincides with the charge of the central extension of susy with $M = Q$

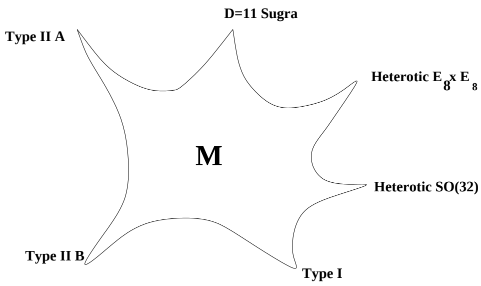

Figure 5.2: M-theory and its perturbative corners.

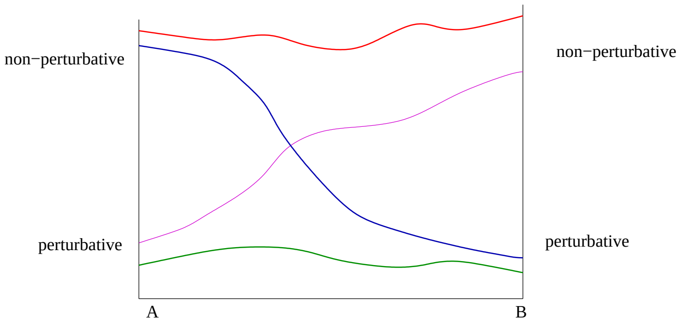

Figure 5.3: As the coupling constant is changed (the dilaton vev, a modulus) the weakly coupled theory A becomes strong coupled but it can be described as weakly interacting theory B. Some perturbative states in A appear as perturbative B states

Thus, the idea that emerges is that there is a unique underlying theory that contains different kinds of BPS p-brane extended objects. Such theory is named M-theory (mother, magic, mysterious?). The different perturbative string theories are just descriptions of such underlying theory in some special regime. This is what is pictorially shown in the, by now even popular, drawing in 5.2.

If one moves from one of these weak couplings regimes (let's say described by theory A) to a strong coupling regime, then the massive solitonic p-brane degrees of freedom can become weakly coupled and thus look as fundamental in this new regime (described by weakly interacting theory B).

This equal footing is referred to as p-brane democracy. The fact that two perturbative corners, namely two apparently different string theories, of M-theory could be connected by continuously changing the string coupling constant is a manifestation of what is called *string duality*.

Since there exist no closed description of the full M-theory the above picture can not be fully proved. Nevertheless, there are several non-trivial indications that support the whole idea. In particular, the identification of BPS states is crucial since they are stable, regardless the value of coupling constant. A pictorial description of the above is presented in fig. 5.3

Just as an example of the present description let us look at the heterotic SO(32) -Type I corners. We have seen in Table I that both theories have a similar massless spectrum. Moreover, the low energy effective action for the heterotic string given in(3.28) reads, in the Einstein frame

$$S_E^{\rm het} = \int d^{10}x\sqrt{g}\left[R - \frac{1}{8}(\nabla\Phi)^2 - \frac{1}{4}e^{-\Phi/4}F^2 - \frac{1}{12}e^{-\Phi/2}\hat{H}^2\right] \tag{5.3}$$

while the Type I effective action in such frame reads,

$$S_E^I = \int d^{10}x\sqrt{g}\left[R - \frac{1}{8}(\nabla\Phi)^2 - \frac{1}{4}e^{\Phi/4}F^2 - \frac{1}{12}e^{\Phi/2}\hat{H}^2\right]. \qquad (5.4)$$

Thus, the two actions are related by $\Phi \to -\Phi$ while keeping the other fields invariant. Since e^Φ is the string coupling constant such relation suggests that the weak coupling of heterotic string is the strong coupling Type I and vice versa. [2]

Recall that the two theories have perturbative expansions that are very different.

Interestingly enough, the D1 string of Type I has the quantum numbers of the $SO(32)$ heterotic fundamental string. The D5 brane of Type I maps into the so called NS5 brane of heterotic.

Similar considerations are valid for other corners of the diagram.

It is important to stress that duality relations span an intricate web when compactifications to lower dimensions are considered.

[2] The fact that the two actions are related by a field redefinition is not a surprise. It is known that $N = 1$ ten-dimensional supergravity is completely fixed once the gauge group is chosen. It is interesting though, that the field redefinition here is just an inversion of the ten-dimensional coupling.

Chapter 6

Brane worlds

In the mid-eighties a new way to look at Particle Physics phenomenology, from a *string theory* point of view, emerged. The main reason for the development of a plausible *String Phenomenology* was the fact that all the ingredients required to embed the observed standard model (SM) physics inside a fully unified theory with gravity were, in principle, present in string theory. Before the so called "duality revolution" the standard framework considered compactifications of the heterotic string. By starting, for instance, with the $E_8 \times E_8$ in $D = 10$ dimensions, compactifications allowed a reduction of the number of dimensions, supersymmetries and the gauge group leading to a massless spectrum as similar as possible to the SM. The guide lines of this approach were presented in Candelas et al. [18] where compactifications on a particular type of manifolds, the Calabi-Yau manifolds, were considered. Other constructions using compact orbifolds or fermionic string models followed essentially the same strategy [19].

We have briefly discussed the compactification idea. Strings propagating in ten dimensional curved X_{10} space-times backgrounds that look like a Minkowski four dimensional spacetime times an internal compact 6-dimensional manifold

$$X_{10} = M_4 \times X_6 \tag{6.1}$$

must be considered. X_6 must be adequately chosen in order to produce a spectrum close to the Standard Model or some extension of it. In particular, the number of fermionic generations is related to the topology of this manifold.[1] For instance, if compactification on a six torus is considered, the states of the ten dimensional theory must be reexpressed in terms of $D = 4$ Poincare representations. However, since the torus is a trivial flat manifold, it is easy to see that no fermion is projected out. The same number of left and right fermions (in the same gauge representation) appear in $D = 4$ and, thus, the theory is non chiral. Calabi-Yau manifolds are such type of non trivial manifold that, moreover, ensure that one supersymmetry is preserved in the compactification process. One important consequence of perturbative heterotic string compactifications is that the string scale is close to the Planck scale, as we discussed in section (3.3).

We will not follow this road here but rather present new scenarios in the context of p-branes.

By now it must be clear that the different classes of p-branes (e.g. D-branes) play a fundamental role in the structure of the full theory of strings. In particular, a fundamental fact is that branes localize gauge interactions on their worldvolume without any need for compactification at this level.

We know that, for example, Type IIB D3-branes have gauge theories with matter fields living in their worldvolume. These fields are localized in the four-dimensional world-volume, and their nature and behaviour depends only on the local structure of the string configuration in the vicinity of that four-dimensional subspace.

[1] Zero modes of the Dirac equation on a non trivial X_6 "box".

Thus, as far as gauge interactions are concerned, a sensible approach should be to first look for *localized* D-brane configurations with world volume field theories resembling as much as possible the SM field theory, even before any compactification of the six transverse dimensions . Our world (SM) would thus be described by a set of branes sitting at some particular point (if Dp-branes, with $p > 3$ are considered the extra dimensions must be compactified) we call it *brane world*.

However the problem of chirality arises also here. Namely, when branes sit a smooth point the spectrum is non -chiral. For instance, open string massless fermionic (Ramond) states $|a\rangle\lambda^a$ on a D3-brane are represented by

$$|s_{st}, s_1, s_2, s_3\rangle\lambda^a \tag{6.2}$$

where $s_{st}, s_i = \pm\frac{1}{2}$ (an odd number of minus signs). $s_{st} = -\frac{1}{2}$ is a negative chirality spinor while $s_{st} = \frac{1}{2}$ corresponds to a positive chirality one. Thus, we have four left and four right chiral fermions. The theory is non-chiral. Actually, an $\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory is obtained if N D3-branes sit on the top of each other.

This result can be easily interpreted. If branes sit on a smooth point we have no obstruction in separate them continuously apart. States arising from branes stretching between two different stacks of branes will become massive. Something that would be forbidden if the theory were chiral.

Essentially two approaches have been followed in order to obtain chiral theories:

1. **Branes at singularities**
2. **intersecting branes**

6.1 branes at singularities

When a stack of D3-branes sits at a singular point some of the possible fermion states are projected out (see i.e. [20] and references therein). For instance, if a C_3/Z_N Z_N orbifold like singularity is considered, states do transform under a Z_N orbifold rotation. If θ is the generator associated to such rotations, its action on a spinor state reads

$$\theta|s_{st}, s_1, s_2, s_3\rangle = e^{2i\pi s_i \frac{a_i}{N}}|s_{st}, s_1, s_2, s_3\rangle \tag{6.3}$$

where a_i are integers (which are, generically, further restricted). Thus, if such a state is required to be invariant under the Z_M action then

$$s_i\frac{a_i}{N} = 0 \quad mod \quad integer \tag{6.4}$$

and some fermions are projected out. This is the basic mechanism that leads to chirality.

Actually, when open string states are considered Z_M action manifests, not only, on world sheet degrees of freedom but also on Chan-Paton factors λ^a. The twist action is represented by a unitary $N \times N$ matrices γ_θ. Thus, a θ twist on an open string state leads to

$$\theta(\Psi_a\rangle\lambda^a)) = (\theta\Psi_a)\rangle\gamma_\theta\lambda^a)\gamma_\theta^{-1} \tag{6.5}$$

For a gauge boson, with space-time indices only, world sheet rotation is trivial and, therefore, invariance of the open string state under orbifold action leads to the restriction

$$\lambda^a) = \gamma_\theta\lambda^a)\gamma_\theta^{-1} \tag{6.6}$$

Namely, the original gauge group $U(N)$, encoded in hermitian Chan-Paton matrices λ^a is broken. A generic consistent γ_θ generically leads to the breaking

$$U(N) \to U(n_0) \times \ldots \times U(n_{M-1}) \tag{6.7}$$

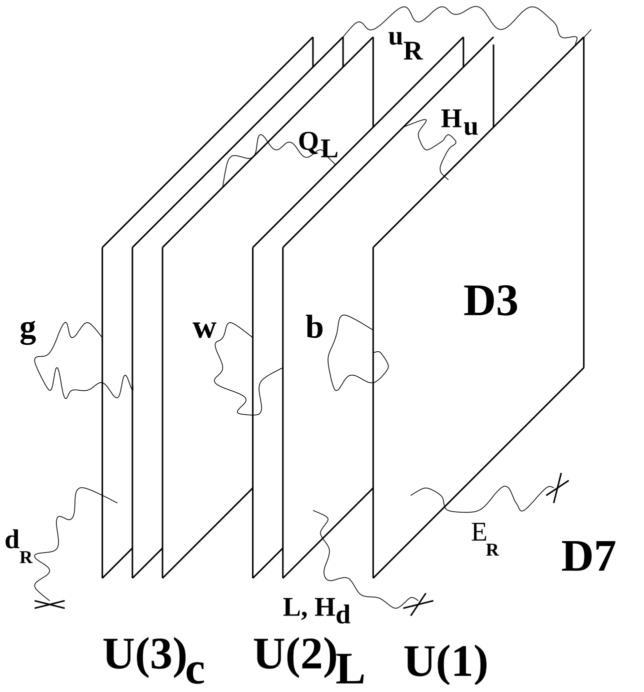

Figure 6.1: D-brane configuration of a SM $\mathbb{Z}_3$ orbifold model. Six D3-branes (with worldvolume spanning Minkowski space) are located on a $\mathbb{Z}_3$ singularity and the symmetry is broken to $U(3)\times U(2)\times U(1)$. For the sake of visualization the D3-branes are depicted at different locations, even though they are in fact on top of each other. Open strings starting and ending on the same sets of D3-branes give rise to gauge bosons; those starting in one set and ending on different sets give rise to the left-handed quarks, right-handed U-quarks and one set of Higgs fields. Leptons, and right-handed D-quarks correspond to open strings starting on some D3-branes and ending on the D7-branes (with world-volume filling the whole figure).

with $n_0 + \ldots + n_{M-1} = N$.

By consistently choosing the eigenvalues $a_i (i = 1, 2, 3)$ an $\mathcal{N} = 1$ supersymmetric, chiral theory can be obtained.

As an interesting example consider $(a_1, a_2, a_3) = (1, 1, -2)$ and a Z_3 action. The following, chiral, spectrum is obtained

$$U(n_0) \times U(n_1) \times U(n_2) \tag{6.8}$$

$$3[(n_0, \bar{n}_1, 1) + (1, n_1, \bar{n}_2) + (\bar{n}_0, 1, n_2)] \tag{6.9}$$

with 3 generations of chiral fields. The 3 is associated to Z_3. Recall that $n_0 = 3, n_1 = 2, n_2 = 1$ would look quite close to Standard Model.

However, it is worth noticing that we have considered just part of the whole construction and different consistent conditions, associated to tadpole cancellation must be imposed. In model above such conditions imply $n_0 = n_1 = n_2$. Interestingly enough these are the conditions that ensure that non chiral anomalies are present. The model is not phenomenologically relevant.

Nevertheless, more sophisticated models are available, for instance, when D7- branes containing the orbifold singularity are present, that lead to much interesting models.

A pictorial representation of a Standard Model like construction is given in figure 6.1.

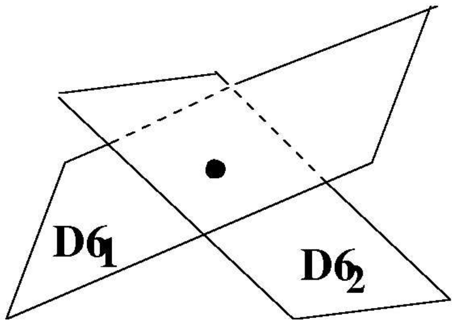

Figure 6.2: .

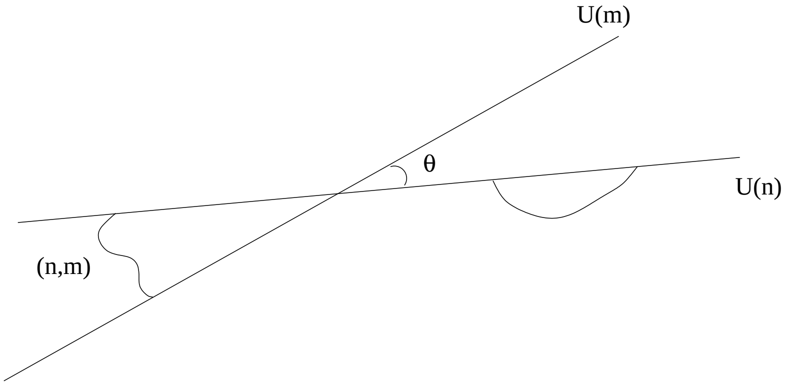

Figure 6.3: Chiral fermions appear at intersections.

6.2 Intersecting branes

There exists another kind of brane configurations that may lead to chiral families. This is the case when branes intersect at angles see [21] and references therein.). Such kind of configuration is shown in figure 6.2. where two D6-branes (Type IIA branes) world volumes intersect over a four dimensional space-time. More general cases will involve several stacks of branes intersecting at different angles. In fact, the geometry of the configuration is encoded in such angles. Open strings with both end-points at the same stack of N branes will lead to $U(N)$ gauge bosons. Interestingly enough, strings stretching between two different stacks can lead to chiral fermions (see fig.6.3). Computation of string modes and quantization proceeds as in previous cases. The difference arises in the boundary conditions.

For, lets say k stacks of branes the gauge group and fermionic matter (left chiral here)

$$\prod_{a}^{k} U(n_a) \tag{6.10}$$

$$\sum_{a<b} I_{ab}(n_a, \bar{n}_b) \tag{6.11}$$

where n_a is the number of branes in stack a whereas I_{ab} is the intersection number which counts the number of times that branes in stack a intersect branes in stack b. This number is not necessarily one (or zero) if branes wrap around cycles on compact dimensions.

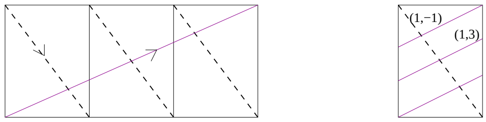

Figure 6.4: Two stacks of branes intersect on a T^2 torus. First stack wraps once on each torus cycle (one with negative orientation), denoted by $(1,-1)$ while the second one wraps once on e_1, vertical direction and three times on horizontal direction e_2, denoted by $(1,3)$. Intersection number is thus $I_{12} = 4$

$N_1 = 3$	(1,0)	(1,-1)	(1,-1)
$N_2 = 2$	(2,1)	(1,2)	(1,0)
$N_3 = 1$	(2,1)	(-1,-2)	(1,0)
$N_4 = 1$	(1,0)	(1,-1)	(-2,2)
$N_5 = 1$	(1,0)	(1,-1)	(-1,1)
$N_6 = 1$	(2,1)	(-1,-2)	(1,0)

Table 6.1: Wrapping numbers

Interestingly enough, in this picture, the number of families is associated to the number of times that cycles intersect in the compact manifold, as shown in figure 6.4. Let us present an explicit interesting example of a Standard like model. Consider a toroidal compact manifold $X_6 = T^2 \times T^2 \times T^2$ defined as factorized two dimensional tori.

Assume, for simplicity that each torus T_i^2 with $i = 1, 2, 3$ is described by two unit vectors (e_1^i, e_2^i). We indicate with (n_a^i, m_a^i) the number of times that stack a wraps around each cycle. Take six stacks of D6 branes with $N_1 = 3$, leading to QCD group, $N_2 = 2$ and $N_3 = N_4 = N_5 = N_6 = 1$. Wrapping numbers are given in Table 6.2 and lead to the following, non-zero, intersection number

$$\begin{aligned}
I_{12} &= 3 = I_{56} = I_{25} \\
I_{13} &= I_{16} = I_{35} = -3 \\
I_{24} &= I_{46} = 6 = -I_{34}
\end{aligned} \tag{6.12}$$

By recalling eq.(6.11) we find the gauge group

$$SU(3) \times SU(2) \times U(1)_Y (\times U(1)'s) \tag{6.13}$$

with the chiral fermion spectrum

$$\begin{aligned}
&3(3, 2, 1/6)_{(1,-1,0,0,0,0)} + 3(\bar{3}, 1, -2/3)_{(-1,0,1,0,0,0)} + 3(\bar{3}, 1, 1/3)_{(-1,0,0,0,0,1)} + \\
&3(1, 2, 1/2)_{(0,1,0,0,-1,0)} + 3(1, 2, -1/2)_{(0,1,0,\underline{-1,0},0,0)} + \\
&3(1, 1, 1)_{(0,0,0,\underline{1,0},0,-1)} + 3(1, 1, -1)_{(0,0,-1,0,1,0)} + \\
&3(1, 1, 0)_{(0,0,0,0,1,-1)} + 3(1, 1, 0)_{(0,0,-1,\underline{1,0},0,0)}
\end{aligned}$$

where underlining means permutation (notice multiwrapping on third torus for N_4). Hypercharge is defined as a linear combination of $U(1)$ generators

$$Y = -(\frac{Q_1}{3} + \frac{Q_2}{2} + Q_5 + Q_6) \tag{6.14}$$

where Q_a is the $U(1)$ generator in $U(N_a)$.

6.3 Scales

In compactifications where D-branes are present the propagation of gauge particles is confined to the low dimensional world volumes of the branes while gravitational interactions propagate in the bulk, ten dimensional space.

This separation of bulk and wrold volume interactions has very important consequences on the string scale.

Consider a $D = 10$ dimensional space is compactified to $D = 4$ dimensional space $M_4 \times X_6$. Let us assume that the gauge sector propagates on the world volume of a Dp-brane while gravity propagates in 10d bulk. Three spatial dimensions of the brane will fill M_4 while the rest will be wrapped on a $p - 3$ internal cycle (manifold). Before compactification to four dimensions the structure of the effective action reads

$$S^{eff}_{d=10} \propto \int d^{10}x \frac{M_S^8}{g_s^2} [R_{10} + \ldots] + \int d^{p+1}x \frac{M_S^{p+1}}{g_s} F^2 \,. \tag{6.15}$$

thus, when comapctification on X_6 is considered, following the steps as above, each integral acquires a different volume factor. Namely,

$$S^{eff}_{d=4} \propto \int d^4x \frac{M_S^8 V_6}{g_s^2} [R_4 + \ldots] + \int d^{p+1}x \frac{M_S^{p+1} V_{||}}{g_s} F^2 \,. \tag{6.16}$$

Thus, we can express the Planck mass and gauge coupling constant in $D = 4$ dimensions in terms of string scale and coupling constants

$$M_P = \frac{M_S^8 V_6}{g_s^2} \simeq 10^{19} GeV \tag{6.17}$$

$$g_{YM} = \frac{M_S^{p+1} V_{||}}{g_s} \simeq 0.1 \tag{6.18}$$

Thus, if $V_6 = V_{||} V_\perp$ then

$$M_P g_{YM} = \frac{M_S^{11-p} V_\perp}{g_s} \tag{6.19}$$

We see that a large Planck mass, with a consistent Yang-Mills coupling constant is obtainable even for low string scales provided the compactification volume is sufficiently large.

In this picture large extra dimensions could be detected as deviations from Newton's inverse square law for the gravitational force, which is valid only if $d = 4$. Actually, due to the small value of Newton's coupling constant, it is very difficult to measure gravitational force at low distances for small masses. From experimental results it is known that inverse square law is valid down to $1/10mm$. A violation at shorter distances would be consistent with experimental bounds.

Interestingly enough scales down to the electroweak scale, of the order of $1TeV$, can be considered. Recall that in this framework no hierarchy problem would arise. However, the problem is transmuted into a geometrical one of explaining why compactification lengths should stabilize at such large values.

These interesing results have given new impetus to experiments aiming to detect deviations from Newton law. Most of them are based on modern technique reformulations of the Cavendish null experiment. Also, if compactification radii are large massive Kaluza Klein replicas of the graviton, for instance, could be detected at accelerators.

Nevertheless, it is important to keep in mind that such low TeV scale is not compulsory but just a cosnsistent possibility. Much larger energy scales are also possible and therefore with a lower chance of direct detection of strings in the near future.

Bibliography

[1] M. Carena, talk at this SILAFAE school.

[2] C. Wagner, talk at this SILAFAE school.

[3] E. Ma, talk at this SILAFAE school.

[4] E. Roulet, talk at this SILAFAE school.

[5] E. Roulet, *Beyond the standard model*, hep-ph/0112348

[6] Particle Data Group
http://pdg.lbl.gov/

[7] Juan M. Maldacena, *Gravity, particle physics and their unification*, Int.J.Mod.Phys.A15S1:840-852,2000. F. Quevedo, *Lectures on superstring phenomenology*, hep-th/9603074
L. Ibañez, *Recent development in physics beyond the Standard Model*, hep-ph/9901292.

[8] H. Davoudiasl, R. Kitano, T. Li, H. Murayama, *The New Minimal Standard Model*, hep-ph/0405097

[9] M. Dine, Tasi Lectures on M theory Phenomenology, hep-th/0003175

[10] J. Polchinski *String Theory* Vol.I y II;Cambridge Univ. Press, 1998

[11] M. Green, J. Schwarz and E. Witten,*Superstring Theory* Vol.I y II;Cambridge Univ. Press, 1987

[12] A. Uranga,*Notes on String Theory*
http://gesalerico.ft.uam.es/paginaspersonales/angeluranga/index.html

[13] E. Kiritsis, *Introduction to Superstring Theory* hep-th/9709062.

[14] G. Aldazabal, *$D = 4$ orientifold models* . Lecture notes of Latinamerican School on Strings, Mexico(October-November 2000).
Atish Dabholkar , *Lecture notes on orientifolds and duality.* hep-th/9804208.

[15] A. Sen, *Tachyon Dynamics in String Theory*, 17th Nishinomiya-Yukawa Memorial Symposium: String Theory, Nishinomiya, Japan, 12-13 Nov 2002. Prog.Theor.Phys.Suppl.152:80-92,2004, Pramana 62:403-413,2004

[16] C. M. Hull, P. K. Townsend, *Unity of superstring dualities*, Nucl. Phys. B438 (1995) 109, hep-th/9410167.

[17] E. Witten, *String theory in various dimensions*, Nucl. Phys. B443 (1995) 65, hep-th/9503124.

[18] P. Candelas, G. Horowitz, A. Strominger, E. Witten, Nucl. Phys. B258 (1985) 46.

[19] Some possible reviews on string phenomenology with reference to the original literature are:
F. Quevedo, hep-ph/9707434; hep-th/9603074 ;
K. Dienes, hep-ph/0004129; hep-th/9602045 ;
J.D. Lykken, hep-ph/9903026; hep-th/9607144 ;
M. Dine, hep-th/0003175;
G. Aldazabal, hep-th/9507162 ;
L.E. Ibáñez, hep-ph/9911499;hep-ph/9804238;hep-th/9505098;
Z. Kakushadze and S.-H.H. Tye, hep-th/9512155;
I. Antoniadis, hep-th/0102202;
E. Dudas, hep-ph/0006190;
D.Bailin, G. Kraniotis, A. Love, *Searching for String theories of the Standard Model*, hep-th/0108127.

[20] G. Aldazabal, L. E. Ibáñez, F. Quevedo, A. M. Uranga, *D-branes at singularities: A Bottom up approach to the string embedding of the standard model* JHEP 0008 (2000) 002, hep-th/0005067.

[21] M. Berkooz, M. R. Douglas, R. G. Leigh, *Branes at angles*, Nucl.Phys. **B480**:265-278,1996; hep-th/9606139.
G. Aldazabal, S. Franco, Luis E. Ibanez, R. Rabadan, A.M. Uranga, *Intesecting brane worlds.* JHEP **0102**:047,2001; hep-ph/0011132.