level with a new combination of weights. The difference in the two sets of weights gives the weight of the unknown object. If we wish to determine the specific gravity of an object we can place the object on the pan P' under water and rebalance the system. The difference between this weighing and the previous one gives the loss of weight in water or the weight of an equivalent volume of water. If the weighings in the order described are W_1, W_2, and W_3, then

$$\text{sp. gr.} = \frac{W_1 - W_2}{W_3 - W_2}.$$

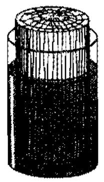

FIG. 14-16 Floating a Battleship in a Quart of Water. A very large block of wood will float in a very small amount of water, if the container and block are about the same volume.

We might ask "Does it take 10,000 tons of water to float a 10,000 ton ship?" The answer is no, and in fact it would be possible to float the ship in a quart of water. Fig. 14-16 shows a block of wood which just fits into a container. A very small amount of water will bring the wood into a floating position. The weight of the displaced water is still equal to the weight of the block but the amount of water needed to do this may be much less or "the block is floating by water that isn't there!"

A variable buoyancy device known as a Cartesian diver, shown in Fig. 14-17, can easily be constructed. A small vial V open at the bottom is inverted

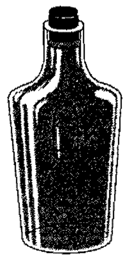

FIG. 14-17 Cartesian Diver. The vial can have its buoyant force varied by compressing the trapped air at its top.

with an air bubble trapped at the top which is just large enough to make the vial float in water. If this vial is in a larger bottle of water which can be closed off with a cork we can vary the pressure by depressing or releasing the cork or squeezing the bottle. With high pressure the volume of trapped air in the vial decreases and the vial sinks. When the pressure is low the air in the vial expands and increases the buoyant force on the vial so that it rises.

14.4 Body Forces in Accelerated Systems

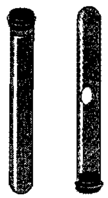

FIG. 14-18 No Gravitational Pressure in a Freely Falling System. Air bubbles will not rise while the system is freely falling.

If a fairly long tube is filled with water except for a small air space at the top and then corked, the air space is buoyed to the top of the tube (Fig. 14-18). When the tube is inverted the bubble rises quickly to the top end. If the tube is tossed into the air while the bubble is rising through the liquid, the rising motion of the bubble stops and the buoyant force seems to disappear. The gravitational forces are now used to produce the acceleration g and there are no pressure forces. During free fall or orbital flight liquids can exert no gravitation pressure or buoyant force.

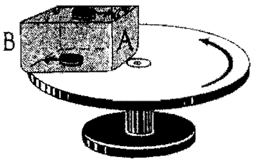

FIG. 14-19 Pressure Produced by Central Acceleration. Objects lighter than the liquid move to the inside edge while objects heavier than water move out.

Suppose a covered glass container is filled with water and is placed on a rotating table as shown in Fig. 14-19. Also suppose an object heavier than water and an object lighter than water are placed in the container. If the table is rotated the heavy object moves outward to face B and the light object moves inward to face A. This happens because a radial pressure change is set up to produce the central acceleration. Cream separators work on this principle. Mercury and water when placed in a rotating bowl as shown in Fig. 14-20 will give a band of mercury around the outside and a band of water inside this.

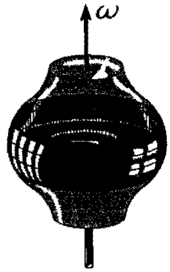

FIG. 14-20 Liquid Separator. A mixture of mercury and water separates with the mercury on the outside when the system is rotated.

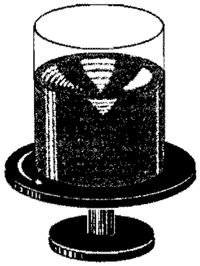

FIG. 14-21 Free Surface of a Rotating Liquid. The top surface becomes a paraboloid of revolution when the beaker is rotated.

If a beaker of liquid is rotated about its axis (Fig. 14-21) eventually viscous forces make the surface become a paraboloid of revolution. In this case the pressure change which is normal to the surface combines with the gravitational force to produce the central acceleration $\omega^2 r$. On a unit volume at the surface (Fig. 14-22), the forces must combine so that $\rho\omega^2 r/\rho g = \tan\theta$. Also $\tan\theta = dy/dr$ is the slope of the curve representing the surface of revolution, so $g\,dy = \omega^2 r\,dr$. The curve giving the surface of revolution can be obtained by integrating this to give $y = \omega^2 r^2/2g$ which is a parabola.

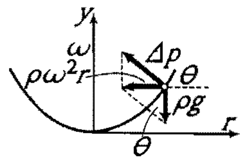

FIG. 14-22 Force Acting at a Rotating Surface. The weight and pressure gradient combine to produce the central acceleration.

14.5 Atmospheric Pressure

In Section 14.2 we showed how the atmosphere exerts a pressure of approximately 10^5 newtons/m^2, i.e. it will support a column of mercury about 76 cm high. Two hemispheres of a few inches radius with carefully ground and fitted edges can be evacuated so that the atmospheric pressure exists only on the outside (Fig. 14-23). A very large force is required to pull the hemispheres apart. Similarly if a gallon can is sealed off except for a connection to a vacuum pump and then evacuated, the air pressure on the outside will easily crush the can (Fig. 14-24). On a square foot of surface, the atmospheric force is nearly one ton.

In the case of air at constant temperature, the density is proportional to the pressure, that is $\rho = kp$ or ρ/p is a constant. The variation of pressure

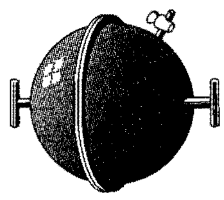

FIG. 14-23 Magdeburg Hemispheres. When the two hemispheres are fitted together and evacuated a very large force is required to pull them apart.

with height in a gravitational field now becomes $dp = -\rho g\, dh = -kpg\, dh$. The integral of this is

$$\int_{p_0}^{p} \frac{dp}{p} = -kg \int_0^h dh \quad \text{or} \quad \ln \frac{p}{p_0} = -kgh.$$

In the exponential form this becomes $p = p_0 e^{-kgh}$ where p_0 is sea level pressure. Later when we study thermodynamics we will evaluate the constant k, but for now we will merely state a convenient way to use this law. If $h = 1/kg = H$, then $p = p_0/e$ or the pressure has decreased to a value $1/e$ of sea level pressure at the height H. For the earth $H = 26{,}200$ ft $= 7.99 \times 10^3$ m at 0°C and is called the characteristic height of the atmosphere. If the law of atmospheres is then written as $p = p_0 e^{-h/H}$, we find that each time h increases by an additional amount H the pressure decreases by a factor of e (Fig. 14-25).

All the phenomena of buoyancy occur in gases just as in liquids. If a hollow metal sphere is balanced with a solid weight at atmospheric pressure the hollow object will displace more air than the solid weight. If the balanced system is then placed under a bell jar and evacuated, the system will become unbalanced with the sphere being "heavier" (Fig. 14-26).

Dirigibles and balloons are supported by the buoyant forces of air, that is, they are buoyed up by a force equal to the weight of the air they displace. An evacuated balloon would be more buoyant but we cannot make them structurally strong enough to withstand the large forces arising from atmospheric pressure. We can, however, fill them with a low density gas such as

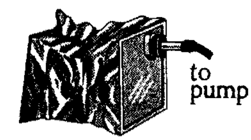

FIG. 14-24. Atmospheric Pressure. When the pressure is lowered on the inside of the can the atmospheric pressure crushes the can.

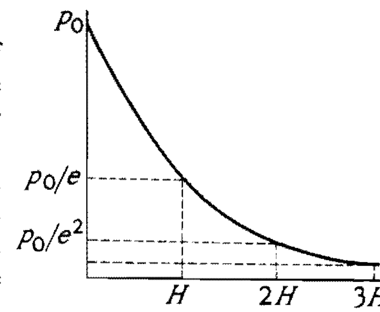

FIG. 14-25 Law of Atmospheres. In an isothermal atmosphere the pressure decreases by a factor of $1/e$ every time the height is increased by the characteristic height.

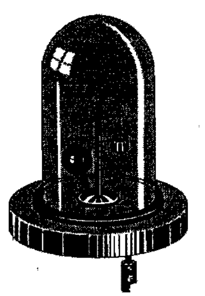

FIG. 14-26 Buoyant Forces of the Atmosphere. When a balanced system is exposed to a vacuum the object with the larger volume appears to become heavier.

hydrogen or helium or even hot air which is capable of exerting pressure and thus keeping the gas bag from collapsing, but adds only a small amount of additional weight.

14.6 Surface Tension

If some narrow-bore glass tubes are placed in a pan of water as shown in Fig. 14-27 the water rises in the tube to some height h above the level of the liquid in the pan. This seems to violate the hydrostatic principle that water seeks its own level. On inspecting the surface of the liquid in the tube we find that it is curved, or we say it has a concave meniscus. We know that there is one atmosphere of pressure p_0 above the liquid in the tube and above the liquid in the bottom pan. In going from the surface of the pan up the bore of the tube to point A just below the meniscus, the pressure drops from

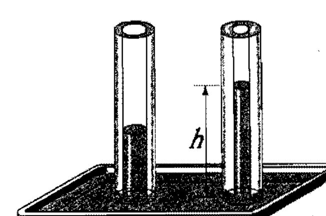

FIG. 14-27 Capillary Tubes. Water rises in small open tubes and the height is inversely proportional to the diameter of the tube.

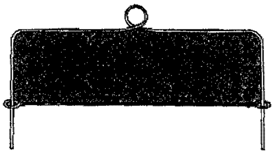

FIG. 14-28 Surface Tension. The stretched film tends to contract indicating that the energy decreases when the surface area decreases.

p_0 to $p_0 - \rho gh$ where ρ is the density of the liquid. Across the curved surface there must be a discontinuous pressure change of ρgh in order to get back to the pressure p_0 just above the meniscus.

Raindrops or any splashed water tends to form spherical drops. We must "blow up" a soap bubble which implies a higher pressure exists inside the bubble than on the outside. Why is this true?

If a wire linkage is made as shown in Fig. 14-28 and covered with a soap film, the film will contract and raise the movable wire. If a soap film is formed across a loop containing a thread as shown in Fig. 14-29a and then

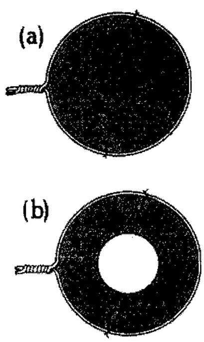

FIG. 14-29 Stretching of Threads Imbedded in Surface Films. When the film is destroyed in any one of the regions the other films contract until the thread becomes taut.

the film is broken within the loop the film pulls the thread into a circular shape as shown in Fig. 14-29b. All these examples indicate that the film acts as a stretched membrane. Why should a liquid behave this way?

A molecule M of a liquid finds itself surrounded by nearest neighbors and is in approximate equilibrium under the forces exerted on it by these nearest neighbors (Fig. 14-30a). The forces are attractive if we try to pull the molecules apart and repulsive if we push them closer together. We note that if a molecule is at the surface the number of nearest neighbors has decreased or that some molecules would have to be pulled away from M in order to obtain this new configuration (Fig. 14-30b). In pulling these molecules away we would have to exert a force through a distance and thus do some work. In other words work must be done to form a liquid surface. Every liquid can have a minimum of energy if its area is a minimum when molecular forces are the predominant forces.

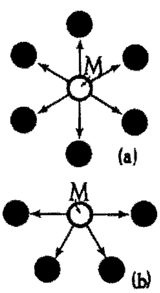

FIG. 14-30 (a) Molecular Forces in Liquids. Within a liquid a molecule M is surrounded by nearest neighbors at an equilibrium distance. Work must be done to push the molecules closer together or to separate them at greater distances. (b) In order for a molecule to be part of a surface some of its nearest neighbors had to be removed which required work to be done on system. Surface tension is the work done per unit area to form the surface.

We define a quantity called surface tension Γ as the free energy per unit area to form the surface. The full significance of the word "free" will have to wait until we study thermodynamics. It will be correct to think of the surface tension as the mechanical work we do to form each unit area of the surface we create. If we take a bent-wire loop and suspend it on a spring as shown in Fig. 14-31 with the loop under water we find that it requires some force to pull the loop through the surface. The significant quantities are shown schematically in Fig. 14-32. Our definition of surface tension gives

$$\Gamma = \frac{Fh}{2hl} = \frac{F}{2l}.$$

The factor 2 enters because we form a total new area of magnitude $2lh$. We note that Γ may also be thought of as being a force per unit length of line drawn in the surface. The force and l can be easily measured. We also note

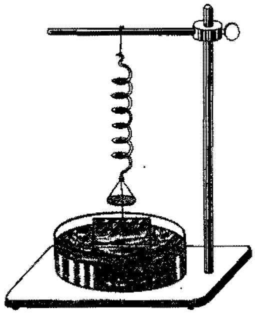

FIG. 14-31 Measurement of Surface Tension. The force per unit length on a wire in the liquid surface can be measured with a calibrated spring.

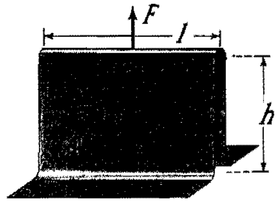

FIG. 14-32 Force on a Wire Breaking through a Surface. The work done to form the additional area $2lh$ is Fh. By definition $\Gamma = Fh/2lh$. The force then is $2l\Gamma = F$.

that we have used a constant force throughout the entire stretching of the distance h of the liquid. This is the correct thing to do and can be verified by returning to the apparatus discussed in Fig. 14-28. It takes no more force to stretch the film through the second centimeter of distance than it does through the first centimeter except for the minute weight of the film. This is quite different from the elastic forces of a spring. A variation of the above experiment is shown in Fig. 14-33 where a buoyant cork is held under water by a wire frame trying to break through the surface.

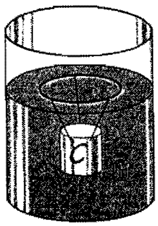

FIG. 14-33 Buoyant Objects can be Held Below a Liquid Surface by Surface Tension.

Many insects that travel on the top of water are supported by surface tension. Razor blades and screen boats float on water because of surface tension forces (Fig. 14-34).

If a small boat with some camphor on its stern is placed on water it will be propelled forward by the surface tension forces (Fig. 14-35). If we think of the stern as a line in the surface, the camphor lowers Γ or the force per unit length in the backward direction. Γ in the forward direction remains larger and pulls the boat forward. Small pieces of camphor dance wildly on a water surface because of unequal surface tension forces around the water line.

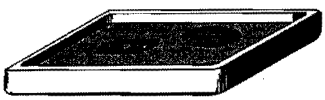

FIG. 14-34 Floating By Surface Tension Forces. Objects more dense than water float if they vary the surface area by a sufficiently large amount.

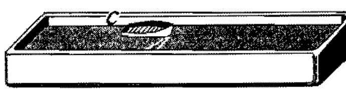

FIG. 14-35 Propulsion by Surface Tension. If the surface tension is different on a section of the periphery of the boat, there will be an unbalanced force on the boat.

14.7 Pressure Changes Due to Surface Tension

A curved liquid surface always behaves as if there is an increased pressure on the concave side. If we consider the wedge shaped volume of liquid

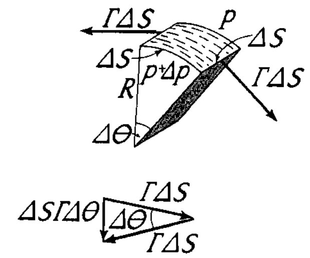

FIG. 14-36 Curved Liquid Surfaces Give Rise to a Pressure Change across the Surface. The curvature allows surface tension forces to have a resultant force directed toward the concave side of the surface.

shown in Fig. 14-36, the two forces of magnitude $\Gamma\Delta s$ each lie in the surface of the liquid and add vectorially to give a force of magnitude $\Gamma\Delta s\Delta\theta$ toward the center of curvature. This must be balanced by the pressure force of $\Delta p(\Delta s)^2$ so $\Gamma(\Delta s)\Delta\theta = \Delta p(\Delta s)^2$. We also see from the geometry that $\Delta\theta = \Delta s/R$ which makes our equation become $\Delta p = \Gamma/R$. If there had been curvature in the other direction as well, our equation for Δp would be $\Delta p = \Gamma(1/R_1 + 1/R_2)$. For a spherical soap bubble there is curvature in both directions of radius R and there are also two surfaces so that $\Delta p = 4\Gamma/R$. If we blow two soap bubbles independently with the apparatus shown in Fig. 14-37 and then connect the two bubbles together, we find that the smaller bubble blows up the larger bubble because it has the smaller radius of curvature and hence the greater internal pressure.

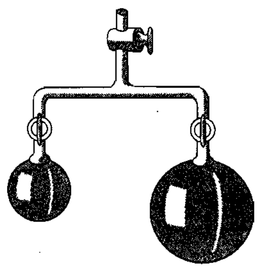

FIG. 14-37 Soap Bubbles. The small soap bubble "blows up" the larger bubble.

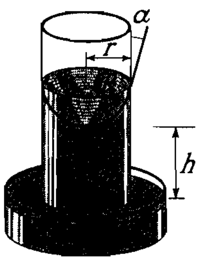

FIG. 14-38 Rise of Water in Capillary Tubes. The change in pressure across the concave meniscus is equal to the pressure change in the column of water.

Let us return to the rise of liquids in the capillary tubes shown in Fig. 14-27. An inspection of the meniscus shows that the surface makes an angle of contact α with the glass walls of the tube (Fig. 14-38). If the radius of the tube is r and the radius of the meniscus is R we have $r = R\cos\alpha$. The change of pressure across the surface is $\Delta p = 2\Gamma/R = (2\Gamma\cos\alpha)/r$. This pressure change must equal the pressure change due to the rise h of the liquid in the tube or $\Delta p = \rho gh$. Equating these two changes in pressure gives $h = (2\Gamma\cos\alpha)/g\rho r$ or the rise is inversely proportional to the radius of the tube. In cases such as mercury on glass, α is greater than 90° so that the mercury is depressed in the tube. When α is less than 90° we say that the liquid wets the surface while if it is greater than 90° the liquid does not wet the surface.

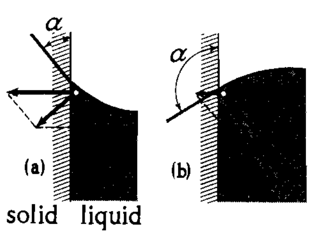

FIG. 14-39 Adhesive and Cohesive Forces. (a) If adhesive forces are greater than cohesive forces the liquid wets the container and gives a concave meniscus. (b) If adhesive forces are greater than cohesive forces the liquid does not wet the surface and the meniscus is convex.

We speak of cohesive forces between like molecules and adhesive forces between unlike molecules. In Fig. 14-39a a case of adhesive forces being greater than cohesive forces is shown so that the resultant force on a molecule near the solid, liquid, and gas surface is directed as shown. The surface of the liquid is a concave meniscus as it tends to form perpendicular to the resultant force on the molecule. The case of greater cohesive forces is shown in Fig. 14-39b.

If two glass plates are stuck together with water (Fig. 14-40a) it requires a large force to pull them apart. The reason for this is that as soon as the plates tend to separate, a meniscus is formed as indicated in Fig. 14-40b. The curvature of the meniscus is such that a lower pressure is created between the plates. Small objects which float on water tend to collect together. This happens for both wetting and nonwetting materials. In either case we find a lower pressure between the objects due to the surface tension effect (Fig. 14-41).

The effects of soap can be seen by floating cork dust on water and then touching the surface with a piece of soap. The particles are suddenly pulled away from the point touched by the soap, indicating a decrease in the surface tension where the soap has touched the surface (Fig. 14-42). Hot water also has a smaller surface tension than cold water, indicating that the molecules become separated further on the average so that the cohesive force between them is smaller. A smaller cohesive force will allow the liquid to spread over a solid surface more readily. Many of the properties of molecules can be inferred from surface tension measurements, but we will have to wait until we study kinetic theory before we can correctly interpret them.

An interesting subject in mathematics is the study of minimal surfaces, and calculation of such surfaces can often be verified with soap films. If we have two wire hoops (Fig. 14-43) with one above the other and ask what is the minimum area that will enclose the region between the two hoops, we find that it is not a cylindrical surface but a curve called a catenoid. If the circular areas are open, there can be no pressure change across the surface. In this case $\Delta p = \Gamma(1/R_1 - 1/R_2) = 0$ so that the radius of curvature in one direction must be equal and opposite to that in the other direction. The energy associated with forming the surface is a minimum when the area of the film is a minimum.

A free drop of liquid such as a rain drop tends to form a minimum surface by being a sphere. If the drop is distorted to an ellipsoidal form it has acquired some potential energy. In returning to a spherical shape it gives kinetic energy to the liquid which can carry the drop to some new distorted shape. The drop oscillates with the characteristic exchange between potential and kinetic energy. A freely falling drop passing through a bright light appears to give regularly spaced streaks since the drop reflects and refracts light differently for the different shapes it has as it oscillates. When rain is observed in bright flood lights it appears to give dashed lines along its trajectory because the light reflected differently for the various configurations of the drop as it oscillates.

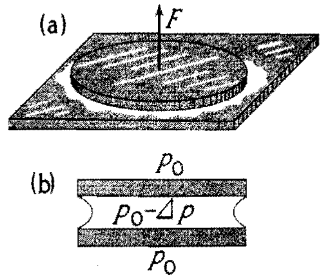

FIG. 14-40 Wet Surfaces Stick Together. Two glass plates are held together by the pressure difference developed across the plates.

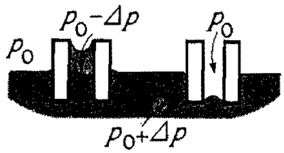

FIG. 14-41 Attraction between Floating Objects. Surface tension tends to reduce the pressure between the small floating objects.

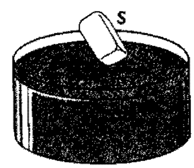

FIG. 14-42 Contracting Surfaces. If sawdust is sprinkled on water and then touched with a bar of soap, the sawdust will rapidly move away from the soap.

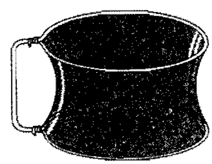

FIG. 14-43 Minimal Surface. The shape of the surface is always adjusted to a minimum area in order to realize a minimum in the energy.