The ontological status of open quantum systems

Sebastian Fortin

CONICET, IAFE (CONICET-UBA) y FCEN (UBA), Argentina

1. Introduction

In the textbooks on quantum mechanics, the laws of the theory are presented as applied to closed quantum systems. The state of a closed quantum system U is represented by a density operator $\hat{\rho}$, and its unitary time evolution is governed by the Schrödinger equation. However, although there is no universally accepted and definitive interpretation of the formalism, certain ideas lead us to carefully consider the concept of quantum system. In fact, the study of phenomena such as relaxation and decoherence requires the introduction of the notion of open system, that is, a quantum system in interaction with other quantum systems.

In general, the subsystems of a closed system U interact with each other. The state of each one of these subsystems is represented by a reduced operator $\hat{\rho}_R$, obtained from the state $\hat{\rho}$ of the total system U by means of the mathematical operation called *partial trace*. The reduced operator $\hat{\rho}_R$ of a subsystem allows us to compute the expectation value of all its observables. For this reason, the usual practice is to conceive open subsystems as legitimate quantum systems (for example, a particle), represented by their corresponding *reduced states* $\hat{\rho}_R$, whose evolution is not ruled by the Schrödinger equation. For this reason, an open quantum system can follow non-unitary evolutions, such as relaxation and decoherence [1].

Decoherence is a process originally proposed to explain the diagonalization of the reduced operator [2]. The orthodox approach considers the system under study embedded in an environment that induces decoherence; then, *environment-induced decoherence* (EID) may only occur in open systems. According to this approach, under certain conditions the reduced state of an open system becomes diagonal, and this fact makes possible its interpretation as a classical state [3]. Thus, decoherence allows us to study the quantum-to-classical transition of a quantum system, for instance, a quantum particle.

In this paper we will study the properties of open systems, and we will discuss their ontological status. First, we will compare the mathematical properties of the quantum state with those of the reduced state. Following the road opened by Bernard d'Espagnat [4], we will argue that, although $\hat{\rho}$ and $\hat{\rho}_R$ have similar mathematical structures, they cannot be interpreted in the same way. In a second stage, we will study the phenomenon of decoherence in situations where the whole closed system can be split into an open system of interest and its environment in different ways [5 - 6]. In particular, we will show that the lack of an univocal criterion to define the open system and its environment is a manifestation of the relative nature of decoherence; this fact prevents us from conceiving the open system as a physical entity of the same ontological status as that of the closed system. Finally, on the basis of previous results [7], we will present a formalism designed to study the phenomena of relaxation and decoherence from a closed-system perspective. Since this formalism does not resort to reduced states, it avoids certain interpretive problems that arise in the orthodox approach. As a consequence, we conclude that, given the problems of interpretation derived from the use of reduced states, the notion of open quantum system should be avoided. According to this viewpoint, the only legitimate quantum system is the whole closed system with its unitary evolution, and the study of its dynamical properties is sufficient to describe the phenomena of decoherence and relaxation.

2. States and expectation values in quantum mechanics

According to the formalism of quantum mechanics, any system has an associated state operator $\hat{\rho}(t)$ that carries all the possible information about the system. The mathematical representation of the state belongs to the Liouville space $\mathcal{L}$. The evolution of the state is given by the Liouville-von Neumann equation [1]:

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}\right] \tag{1.1}$$

where $\hat{H}$ is the Hamiltonian of the system. Every physical property of the system is represented by a specific observable $\hat{O}$ belonging to the dual space of $\mathcal{L}$, $\mathcal{L}'$. In order to compute quantities of physical interest, algebraic operations have to be applied to these operators. For example, the expectation value of the property represented by $\hat{O}$, for a system in state $\hat{\rho}$, is computed as the trace of the product of the two corresponding operators [8]:

$$\left\langle \hat{O} \right\rangle_{\hat{\rho}} = Tr\left(\hat{\rho}\,\hat{O}\right) \tag{1.2}$$

In the case of systems that are not composite, there is no substantial difference between knowing the state and knowing the expectation values of all the observables of the system:

(i) given the state, the expectation value of any observable can be computed, and

(ii) given the expectation value of any observable, the state of the system can be computed.

In this non-composite case, it is usual to work in the context of a state-based description. Nevertheless, it is worth recalling that, given the nature of quantum mechanics, all the information of physical interest is given through expectation values.

In the case of composite systems, the initial state of the whole system is computed as the tensor product of the states of its subsystems. For instance, in a two-particle system, the procedure is the following one [9]:

- We consider that the two particles are initially independent (
- simple systems): particle 1 in the state $\hat{\rho}_1(0)$ and particle 2 in the state $\hat{\rho}_2(0)$.
- We assume that, at a certain time, the two particles begin to interact with each other and, since then, they are considered parts of a composite system whose initial state is $\hat{\rho}_T(0)$.
- We compute the initial state of the composite system by means of the tensor product of the original particle's initial states: $\hat{\rho}_1(0) \otimes \hat{\rho}_2(0) = \hat{\rho}_T(0)$.
- At the initial time, the state of each particle can be recovered by means of the algebraic operation of partial trace, which consists in tracing (deleting) the degrees of freedom of the other particle:

$$\hat{\rho}_1(0) = Tr_2\left(\hat{\rho}_T(0)\right) \quad \text{and} \quad \hat{\rho}_2(0) = Tr_1\left(\hat{\rho}_T(0)\right)$$

- The total state of system evolves, as any quantum state, according to the Liouville von Neumann equation:

$$\frac{d\hat{\rho}_T(t)}{dt} = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}_T(t)\right] \tag{1.3}$$

2.1 The open-system perspective

In many physical problems it is useful to take up the idea of individual particles as the components of the total system. This idea is suggested by the fact that, at the initial time, the partial trace onto the total state of the closed system yields the state of each component particle. However, the generalization of this procedure for all times is, at least, controversial.

Once the total closed system has been identified, it evolves as a whole according to the Liouville-von Neumann equation governed by the total Hamiltonian. Nothing prevents us to take the partial trace of the evolved total state, obtaining a mathematical entity called *reduced state* that has the appearance of a quantum state:

$$\hat{\rho}_1(t) = Tr_2\left(\hat{\rho}_T(t)\right) \tag{1.4}$$

But this reduced state *does not evolve according to the Liouville-von Neumann equation*:

$$\frac{d\hat{\rho}_1(t)}{dt} \neq \frac{1}{i\hbar}\left[\hat{H}_1, \hat{\rho}_1(t)\right] \tag{1.5}$$

The dynamics of the reduced state is ruled by a non-unitary master equation, different in each particular problem. However, it is common to assume that the particle 1 can be reidentified in the composite system, i.e., that it is a quantum system (although not obeying the Liouville-von Neumann equation) represented by the reduced state operator $\hat{\rho}_1(t)$. What supports this assumption is the fact that, although the evolution of the reduced state is not ruled by the Liouville-von Neumann equation, given any observable $\hat{O}_1$ of the particle 1, its expectation value can be computed as:

$$\left\langle \hat{O}_1 \right\rangle_{\hat{\rho}_1(t)} = Tr\left(\hat{O}_1\hat{\rho}_1(t)\right) \tag{1.6}$$

This means that the expectation values of the observables corresponding to particle 1 are computed in the same way as in the case of closed systems, but using the reduced state operator $\hat{\rho}_1(t)$ instead of the total state. Those expectation values are all the information that can be obtained when one has experimental access only to particle 1.

2.2 The closed-system perspective

Although we may be interested in studying the behavior of the different parts of a closed system, the concept of reduced state is controversial, as d'Espagnat showed in his already classical arguments [4]. For this reason, we will show that it is not necessary to resort to reduced states for describing the behavior of the parts of a closed system.

The closed-system perspective allows us to study the evolution of the different degrees of freedom in a closed system by considering the expectation values of the corresponding relevant observables. Indeed, if we consider observables that act only on the system of interest (particle 1):

$$\hat{O}_R = \hat{O}_1 \otimes \hat{I}_2 \tag{1.7}$$

where $\hat{O}_1$ is any observable of particle 1 and $\hat{I}_2$ is the identity of the space of observables of particle 2, then we can compute the expectation values of these observables as follows:

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}_T(t)} = Tr\left(\hat{O}_R \hat{\rho}_T(t)\right) = Tr\left(\left(\hat{O}_1 \otimes \hat{I}_2\right)\hat{\rho}_T(t)\right) = Tr\left(\hat{O}_1 \hat{\rho}_1(t)\right) = \left\langle \hat{O}_1 \right\rangle_{\hat{\rho}_1(t)} \tag{1.8}$$

This last expression tells us that it is not necessary to define the reduced state, nor even to mention the particle 1. We can obtain the expectation values $\left\langle \hat{O}_1 \right\rangle_{\hat{\rho}_1(t)}$, i.e. the information of interest, by studying the behavior of the state of the closed system $\hat{\rho}_T(t)$ and the relevant observables $\hat{O}_R$ of the closed system.

This shows that, in the case of composite systems, there is no difference between knowing the reduced state and knowing the expectation values of all the relevant observables of the form $\hat{O}_R = \hat{O}_1 \otimes \hat{I}_2$:

(i) given the reduced state of the particle, the expectation value of any relevant observable can be computed, and

(ii) given the expectation value of any relevant observable, the reduced state of the particle can be computed.

Therefore, in this case the state-based description of the particle is not necessary: its behavior can be accounted for from the perspective of the whole closed system.

3. Reduced states in decoherence

The Correspondence Principle establishes that it should be possible to recover the laws of classical mechanics from those of quantum mechanics [10]. One way to establish the link between both theories is through the theory of algebraic deformations [11 -12], whereby it is possible to "deform" an algebra to turn it into another, through some operator. By means of this theory it is possible to transform the quantum state $\hat{\rho}$ into a distribution $\rho(q,p)$ in phase space. Physicists aim to interpret this function as a probability distribution in phase space (a state $\rho(q,p)$ of classical statistical mechanics), which sets the probability that the system has a well defined position-momentum classical pair. But for a correct operation of this procedure, it is necessary a diagonal quantum state $\hat{\rho}$. For this reason, decoherence is a process originally designed to explain the diagonalization of the state.

3.1 Environment Induced Decoherence

In the context of the orthodox version, *Environment Induced Decoherence* (EID), the formalism of decoherence applies only to open systems because, as its name suggests, the system *S* under study is considered embedded in an environment *E* which induces decoherence [3]. The system *S* is an open system with an associated Hilbert space $\mathcal{H}_S$, and the environment is an open system with an associated Hilbert space $\mathcal{H}_E$. The corresponding von Neumann-Liouville spaces are $\mathcal{L}_S = \mathcal{H}_S \otimes \mathcal{H}_S$ and $\mathcal{L}_E = \mathcal{H}_E \otimes \mathcal{H}_E$. According to the EID approach, the study of decoherence is based on the study of the evolution of the reduced state represented in a given basis. Either explicitly computing the state $\hat{\rho}_S(t)$ or analyzing case by case the master equation, we can determine whether, under certain conditions, the reduced state operator becomes diagonal or not. The non-diagonal terms of the state are linked to events that do not have a classical analogue. For this reason it is usually said that, when the state becomes diagonal, it represents the classical aspects of the system (to complete the classical limit we must also apply the Wigner transform and $\hbar \to 0$). In many models with a huge number of degrees of freedom, it is shown that:

$$\lim_{t \to t_D} \hat{\rho}_S(t) = \hat{\rho}_S^{(D)}(t) \quad \textit{diagonal} \tag{1.9}$$

According to the EID approach, since after a decoherence time t_D the operator $\hat{\rho}_S(t)$ becomes diagonal $\hat{\rho}_S^{(D)}(t)$, then there is a process of decoherence induced by the large number of degrees of freedom of the environment. This is equivalent to think that $\hat{\rho}_S(t)$ represents the state of a part of the total system, and that this part became classical. In general, this part is interpreted as a particle. Since the state became diagonal and represents a particle, then it is usually said that this particle became classical.

3.2 Environment Induced Decoherence and the measurement problem

The measurement problem is a central interpretive issue for quantum mechanics. When the measurement of a property is performed on a quantum system, although the system is in a superposition state, the reading of the apparatus is a well-defined value. This fact has no adequate explanation in the framework of the theory.

In the standard von Neumann model, a quantum measurement is conceived as an interaction between a system $\mathcal{S}$ and a measuring device $\mathcal{D}$. Before the interaction, $\mathcal{D}$ is prepared in a ready-to-measure state $|r_0\rangle$, eigenvector of the pointer observable R of $\mathcal{D}$, and the state of $\mathcal{S}$ is a superposition of the eigenstates $|a_i\rangle$ of an observable A of $\mathcal{S}$. The interaction introduces a correlation between the eigenstates $|a_i\rangle$ of A and the eigenstates $|r_i\rangle$ of R:

$$|\psi_0\rangle = \sum_i c_i |a_i\rangle \otimes |r_0\rangle \quad \rightarrow \quad |\psi\rangle = \sum_i c_i |a_i\rangle \otimes |r_i\rangle \tag{1.10}$$

The problem consists in explaining why, being the state $|\psi\rangle$ a superposition of the $|a_i\rangle \otimes |r_i\rangle$, the pointer R acquires a definite value.

In the orthodox collapse interpretation, the pure state $|\psi\rangle$ is assumed to "collapse" to a mixture ρ^c:

$$\rho^c = \sum_i |c_i|^2 |a_i\rangle \otimes |r_i\rangle \langle a_i| \otimes \langle r_i| \tag{1.11}$$

where the probabilities $|c_i|^2$ are given an ignorance interpretation. Then, in this situation it is supposed that the measuring apparatus is in one of the eigenvectors $|r_i\rangle$ of R, say $|r_k\rangle$, and therefore R acquires a definite actual value r_k, the eigenvalue corresponding to the eigenvector $|r_k\rangle$, with probability $|c_k|^2$.

The theory of decoherence attempts to reproduce the result expressed by eq. (3.3) but without invoking collapse. According to the usual view, after the interaction between the system and the apparatus, the composite system is in a superposition state:

$$|\Psi\rangle = \sum_{i=1}^{N} c_i |a_i\rangle \otimes |r_i\rangle \tag{1.12}$$

However, this expression only takes into account a single variable of the apparatus, which indicates the position of the pointer. According to EID, this is an extremely idealized simplification: a real measurement instrument is a macroscopic object composed of a huge number of atoms. Then, in a realistic description it is necessary to consider the internal degrees of freedom of the instrument and the interaction with the environment. Therefore, the initial state ($t = 0$) of the composite system is

$$\left|\Psi_{SME}\right\rangle = \sum_{i=1}^{N} c_i \left|a_i\right\rangle \otimes \left|r_i\right\rangle \otimes \left|\varepsilon_0\right\rangle \tag{1.13}$$

Following the usual arguments of decoherence theory [3], we assume that the possible states for the environment E are $\left\{\left|\varepsilon_j\right\rangle\right\}$, and that there is a particular interaction Hamiltonian between system, apparatus and environment. This interaction produces two important effects

- The states of the three systems correlate with each other:

$$\begin{array}{c} \left|a_1\right\rangle \otimes \left|r_1\right\rangle \otimes \left|\varepsilon_0\right\rangle \to \left|a_1\right\rangle \otimes \left|r_1\right\rangle \otimes \left|\varepsilon_1\right\rangle \\ \left|a_2\right\rangle \otimes \left|r_2\right\rangle \otimes \left|\varepsilon_0\right\rangle \to \left|a_2\right\rangle \otimes \left|r_2\right\rangle \otimes \left|\varepsilon_2\right\rangle \\ \vdots \\ \left|a_N\right\rangle \otimes \left|r_N\right\rangle \otimes \left|\varepsilon_0\right\rangle \to \left|a_N\right\rangle \otimes \left|r_N\right\rangle \otimes \left|\varepsilon_N\right\rangle \end{array} \tag{1.14}$$

- The states of the environment become (approximately) orthogonal:

$$\left\langle \varepsilon_i \mid \varepsilon_j \right\rangle \to 0 \tag{1.15}$$

According to the decoherence theory, the interaction produces both effects very fast. Then, the state of the whole system is:

$$\hat{\rho}_{SME} = \left|\Psi_{SME}\right\rangle\left\langle\Psi_{SME}\right| = \sum_{i,j=1}^{N} c_i c_j^* \left|a_i\right\rangle \otimes \left|r_i\right\rangle \otimes \left|\varepsilon_i\right\rangle\left\langle\varepsilon_j\right| \otimes \left\langle r_j\right| \otimes \left\langle a_j\right| \tag{1.16}$$

Since the total system follows a unitary evolution, this state $\hat{\rho}_{SME}$ cannot evolve into a classical state. However, if we take the partial trace over the environmental degrees of freedom, we obtain the reduced state:

$$\hat{\rho}_{SM} = Tr_E\left(\hat{\rho}_{SME}\right) = \sum_{i=1}^{N} \left|c_i\right|^2 \left|a_i\right\rangle \otimes \left|r_i\right\rangle\left\langle r_i\right| \otimes \left\langle a_i\right| \tag{1.17}$$

According to the EID approach, $\hat{\rho}_{SM}$ represents a mixture state containing only the terms corresponding to the classical correlations and, therefore, it can be interpreted in terms of ignorance: the system *SM* is in one of the states $|a_k\rangle \otimes |r_k\rangle$, and the probability $|c_k|^2$ measure our ignorance about the state of the system.

The similarity between the expressions (3.9) and (3.3) suggests that decoherence leads to the same state obtained with the collapse hypothesis. According to Zurek, it is precisely the interaction between the system and its environment what produces an "*illusion of a collapse*" [13]. This suggests that decoherence is able to solve the measurement problem; in the words of Gennaro Auletta, "*Decoherence is able to solve practically all the problems of measurement which have been discussed in the preceding chapters*" [14].

3.3 Traditional problems of Decoherence

As some point out [15 – 16], the theory of environment-induced decoherence has become the "new orthodoxy" in the quantum physicists community. However, the ability of decoherence to solve the problem of measurement has been widely discussed. Whereas many authors consider that decoherence has finally supplied the right answer to the measurement problem (see, *e.g.*, [14] [17]), not all are so enthusiastic: the account of measurement given by the decoherence theorists has been severely criticized on the basis of different arguments (see, *e.g.*, [18 – 19]). Many criticisms point to the implicit assumption that the reduced state obtained in the process of decoherence is equivalent to a classical mixture.

The collapse hypothesis establishes that the state of the system non-unitarily turns into one of the states of the superposition. As a consequence, the system acquires a well-defined value of the measured observable. Therefore, if we repeat the measurement, we can make statistical calculations on systems with well-defined values: this means that, after measurement, the system is represented by a statistical operator as that of the expression (3.3), which is a classic mixture where the probabilities can be interpreted by ignorance. The case of decoherence is completely different, since collapse does not occur but, as Zurek says, the state appears to have collapsed. So, the system does not acquire a well-defined value for the measured observable: the state $|\Psi_{SME}\rangle$ is a superposition at any times. For instance, Adler claims that the diagonalized reduced state $\hat{\rho}_{SM}$ does not allow us to say that the state of the system S is in one of the eigenstates $|r_i\rangle$ of the observable $\hat{R}$, and he concludes: "*I do not believe that either detailed theoretical calculations or* *recent experimental results show that decoherence has resolved the difficulties associated with quantum measurement theory*” ([20], p. 135).

According to Bub [16], if the eigenstate-eigenvalue link is accepted, the problem is even worst: the reduced state is not only unable to explain the occurrence of only one of the eigenvalues r_i of $\hat{R}$, but it is also inconsistent with that occurrence, since the state of the composite system $S+E$ is always the entangled state $|\psi_{SE}\rangle$. Indeed, if we interpret the reduced state $\hat{\rho}_{SM}$ in terms of ignorance we are forced to admit that the observable has definite value. On the other hand, according to the eigenstate-eigenvalue link, an observable has defined value if and only if the system state is an eigenstate of the observable. But without collapse, the state of the entire system is always a superposition of the eigenstates of the observable. So the reduced state $\hat{\rho}_{SM}$ not only fails to account for the occurrence of a single event associated with a definite value of the pointer, but actually is inconsistent with this occurrence.

The origin of this difficulty of EID is rooted in the problem of quantum non-separability. In Schrödinger words “*When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz, by endowing each of them with a representative of its own*” ([21], p. 555). However, EID is based on the features of reduced states, and it intends to describe the open system in the same way as closed systems are described.

Perhaps the clearest (and certainly the most cited) explanation of the difference between the state of a closed system and the state of an open system is that offered by Bernard d'Espagnat in 1966 [22]. He distinguishes between *proper mixture*, the state of a closed system, and *improper mixture*, the state of an open system, which is obtained by tracing off its environment. According to d'Espagnat (see also [4]), although proper and improper mixtures are represented by the same mathematical object, they denote different concepts. The state of the composite system is univocally determined by the reduced state of its components only in the particular case in which there are no correlations between these components. Therefore, if we can only make measurements on the open system of interest, but have no access to the environment, then we would not be able to distinguish between an improper mixture and a proper mixture. But there is no theoretical reason that prevents us from having access to the traced off degrees of freedom. Such an access would allow us to show that the proper mixture and the improper mixture are, in principle, empirically different [4]. On this basis d'Espagnat claims that decoherence can account only for appearances, but not for the ontological realm [23].

These and similar arguments have led even some contributors to the decoherence program to express their skepticism about the relevance of decoherence to the solution of the measurement problem; as Joos ([24], p. 14) says: "*Does decoherence solves the measurement problem? Clearly not*." Here we will argue that a way of understanding this last claim is to notice that, by contrast to Zurek's strategy, the reduced state of the decohering system S must not be conceived as its quantum state.

Moreover, although at present the EID approach has been applied to a wide range of models, and its results have many experimental confirmations (see [24]), it still nevertheless has to face three further conceptual difficulties:

(a) it cannot be applied to closed systems, in particular, to the universe; according to Zurek, the issue of the classicality of closed systems or of the universe as a whole cannot even be posed (see [25], p.181);

(b) it does not supply a criterion for deciding where to place the cut between system and environment; as Zurek himself admits, this is a serious problem for the foundation of the whole EID program (see [26], p.22);

(c) it does not provide a simple general definition of the pointer basis (see [27-28]).

4. Decoherence in the whole and the parts

The solution of the problem of the classical limit of quantum mechanics amounts to accounting how a quantum system behaves as a classical one. Then, if the aim is to explain the classical limit by means of decoherence, it is necessary to identify the systems involved in the decoherence in an objective way. Given a closed system, the application of the EID formalism requires to establish a cut between system and environment. If we consider that the reduced state represents the system that becomes classical, then this state must be unique; otherwise, the objectivity of the classical world would be lost.

In spite of its impressive practical success, from a conceptual viewpoint the EID approach still faces a difficulty derived from its open-system perspective: the problem of defining the system that decoheres. Zurek concedes that this absence of a general criterion to discriminate between system and environment is a serious difficulty of his proposal: "*In particular, one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the "systems" which play such a crucial role in all the discussions of the emergent classicality. This issue was raised earlier, but the progress to date has been slow at best*" (see [26], p.122; for a discussion of this point, see [29]).

In this section we will show the problem in a concrete example. Different cuts between system and environment lead to different classical systems. This shows that the use of the reduced state in the classical limit problem brings new conceptual problems.

4.1 A generalized spin-bath model

This is a generalization of the very simple model that has been exactly solved in previous papers [30] [31] [32] [33] [34]. Let us consider a closed system $U = A \cup B$ where:

(i) The subsystem A is composed of M spin-1/2 particles A_i, with $i = 1,2,\cdots,M$, each one of them represented in its Hilbert space $\mathcal{H}_{A_i}$: in each A_i, the two eigenstates of the spin operator $S_{A_i,\vec{v}}$ in direction $\vec{v}$ are $\left|\Uparrow_i\right\rangle$ and $\left|\Downarrow_i\right\rangle$.

(ii) The subsystem B is composed of N spin-1/2 particles B_k, with $k = 1,2,\cdots,N$, each one of them represented in its Hilbert space $\mathcal{H}_{B_k}$: in each B_k, the two eigenstates of the spin operator $S_{B_k,\vec{v}}$ in direction $\vec{v}$ are $\left|\uparrow_k\right\rangle$ and $\left|\downarrow_k\right\rangle$.

The Hilbert space of the composite system $U = A \cup B$ is, then,

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B = \left(\bigotimes_{i=1}^{M} \mathcal{H}_{A_i} \right) \otimes \left(\bigotimes_{k=1}^{N} \mathcal{H}_{B_k} \right) \tag{1.18}$$

and a pure initial state of U reads

$$\left|\psi_0\right\rangle = \left|\psi_A\right\rangle \otimes \left|\psi_B\right\rangle = \left(\bigotimes_{i=1}^{M} \left(a_i \left|\Uparrow_i\right\rangle + b_i \left|\Downarrow_i\right\rangle \right) \right) \otimes \left(\bigotimes_{k=1}^{N} \left(\alpha_k \left|\uparrow_k\right\rangle + \beta_k \left|\downarrow_k\right\rangle \right) \right) \tag{1.19}$$

with $|a_i|^2+|b_i|^2=|\alpha_k|^2+|\beta_k|^2=1$. As in the original spin-bath model, the self-Hamiltonians H_{A_i} and H_{B_k} of the particles are taken to be zero, and there is no interaction among the particles A_i nor among the particles B_k. As a consequence, the total Hamiltonian $H=H_A\otimes H_B$ of the composite system U is given by

$$H=\sum_{i=1}^{M}\left(\frac{1}{2}\left(|\Uparrow_i\rangle\langle\Uparrow_i|-|\Downarrow_i\rangle\langle\Downarrow_i|\right)\otimes\left(\bigotimes_{\substack{j=1\\ j\neq i}}^{M}I_{A_j}\right)\right)\otimes\left(\sum_{k=1}^{N}\left(g_k\left(|\uparrow_k\rangle\langle\uparrow_k|-|\uparrow_k\rangle\langle\downarrow_k|\right)\otimes\left(\bigotimes_{\substack{l=1\\ l\neq k}}^{N}I_{B_l}\right)\right)\right) \tag{1.20}$$

where $I_{A_j}=|\Uparrow_j\rangle\langle\Uparrow_j|+|\Downarrow_j\rangle\langle\Downarrow_j|$ and $I_{B_l}=|\uparrow_l\rangle\langle\uparrow_l|+|\downarrow_l\rangle\langle\downarrow_l|$ are the identity operators on the subspaces $\mathcal{H}_{A_j}$ and $\mathcal{H}_{B_l}$ respectively. Let us notice that the original model of [30] is the particular case of this generalized model for $M=1$.

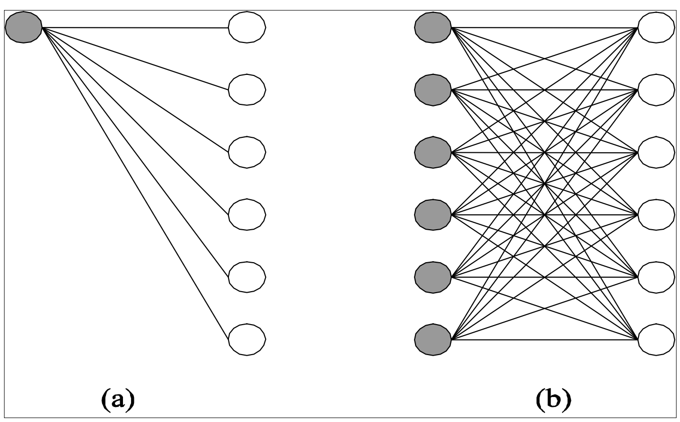

Figure 1. Schema of the interactions among the particles of the open system *A* (grey circles) and of the open system *B* (white circles): (a) original spin-bath model (*M* = 1), and (b) generalized spin-bath model (*M* ≠ 1)

Then, the state $|\psi(t)\rangle$ is obtained as

$$|\psi(t)\rangle=U_t|\psi_0\rangle=e^{-iHt}|\psi_0\rangle=e^{-i(H_A\otimes H_B)t}|\psi_0\rangle \tag{1.21}$$

4.2 Decomposition 1

We can consider the decomposition where A is the open system S and B is the environment E. This is a generalization of the traditional spin-bath model. The only difference with respect to that case is that here the system S is composed of $M \geq 1$ particles instead of only one. Then, here the decomposition is

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E = \left(\bigotimes_{i=1}^{M} \mathcal{H}_{A_i} \right) \otimes \left(\bigotimes_{k=1}^{N} \mathcal{H}_{B_k} \right) \tag{1.22}$$

When the evolution of the reduced state is computed, two cases can be distinguished (see [33]):

- **Case (a):** $M \ll N$

  Numerical simulations show that there is decoherence. This means that, as expected, a small open system $S = A$ of M particles decoheres in interaction with a large environment $E = B$ of $N \gg M$ particles.

- **Case (b):** $M \gg N$ or $M \simeq N$

  Numerical simulations show that there is no decoherence. This means that, when the environment $E = B$ of N particles is not large enough when compared with the open system $S = A$ of M particles, S does not decohere.

4.3 Decomposition 2

In this case we decide to observe only one particle of the open system A. This amounts to splitting the closed system U into two new subsystems: the open system S is, say, the particle A_M and the environment is $E = \left(\bigcup_{i=1}^{M-1} A_i \right) \cup B = \left(\bigcup_{i=1}^{M-1} A_i \right) \cup \left(\bigcup_{k=1}^{N} B_k \right)$. Let us notice that the Decomposition 2 of the traditional spin-bath model is a particular case of this one, for $N = 1$ (where N plays the role of the M of this case). The decomposition here is

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E = \left(\mathcal{H}_{A_M} \right) \otimes \left(\left(\bigotimes_{i=1}^{M-1} \mathcal{H}_{A_i} \right) \otimes \left(\bigotimes_{k=1}^{N} \mathcal{H}_{B_k} \right) \right) \tag{1.23}$$

When the evolution of the reduced state is computed, numerical simulations show that, if $N \gg 1$, there is decoherence. This means that the particle A_M decoheres when $N \gg 1$, independently of the value of M. But since the particle A_M was selected conventionally, the same argument holds for any particle A_i of A. Then, when $N \gg 1$ and independently of the value of M, any particle A_i decoheres in interaction with its environment E of $N+M-1$ particles. On the other hand, the symmetry of the whole system U allows us to draw analogous conclusions when the system S is one of the particles of B: when $M \gg 1$ and independently of the value of N, any particle B_k decoheres in interaction with its environment E of $N+M-1$ particles.

4.4 Analyzing results

Let us consider the generalized spin-bath model when $M \simeq N \gg 1$. In this case, the subsystem $A = \bigcup_{i=1}^{M} A_i$ does not decohere (Decomposition 1), but the particles A_i, considered independently, do decohere (Decomposition 2). In other words, in spite of the fact that certain particles decohere and may behave classically, the subsystem composed by all of them retains its quantum nature. We have also seen that, since $M \gg 1$, all the particles B_k, considered independently, decohere. Then, in this case not only all the A_i, but also all the B_k decohere. This means that all the particles of the closed system $U = \left(\bigcup_{i=1}^{M} A_i\right) \cup \left(\bigcup_{k=1}^{N} B_k\right)$ may become classical when considered independently, although the whole system U certainly does not decohere and, therefore, retains its quantum character.

The fact that certain particles may be classical or quantum depending on how they are considered sounds paradoxical in the context of an approach that explains decoherence as the result of an interaction between open systems. This apparent paradox is rooted in the interpretation of the reduced state as a legitimate quantum state. The open-system approach not only leads to the problem of deciding where to place the cut between system and environment, but in a certain sense also disregards the well-known holism of quantum mechanics: a quantum system in not the mere collection of its parts and the interactions among them. In order to retain its holistic nature, a quantum system has to be considered as a whole. For this reason, in the next section we introduce a description of decoherence from the closed-system perspective

5. The study of expectation values as an alternative to the study of the state in EID

Although the EID approach is based on the study of the diagonalization of the reduced state, it is also possible to analyze the quantum-to-classical transition at the level of expectation values. For simplicity, we will consider the case of systems with discrete energy spectrum; nevertheless, the same considerations can be done in the continuous case.

According to classical statistics, we can compute the expectation value of an observable O by summing the possible values of this observable weighed with probability of being measured. If those possible values are $o_1, o_2, ..., o_N$, $\Omega = \{o_1, o_2, ..., o_N\}$ is the space of possible events, and P_i is the probability to measure o_i, then the expectation value is

$$\langle O \rangle = \sum_i o_i P_i \tag{1.24}$$

However, the expectation values of quantum mechanics have a different structure. The expectation value of an observable $\hat{O}$ in the state $\hat{\rho}$ is given by

$$\langle \hat{O} \rangle_{\hat{\rho}} = \sum_{i,j} O_{ij} \rho_{ji} \tag{1.25}$$

In this expression we can distinguish the term resulting from the diagonal components of $\hat{O}$ and $\hat{\rho}$, and the term resulting from the non-diagonal components:

$$\langle \hat{O} \rangle_{\hat{\rho}} = \sum_i O_{ii} \rho_{ii} + \sum_{i \neq j} O_{ij} \rho_{ji} \tag{1.26}$$

The diagonal elements $O_{ii} = O_i$ of the observable $\hat{O}$ appear in the first sum, which could be interpreted as values to be measured multiplied by the diagonal elements ρ_{ii} of the state operator, that precisely correspond to the probabilities $P_{ii} = P_i$ assigned by the Born rule. That is,

$$\langle \hat{O} \rangle_{\hat{\rho}} = \sum_i O_i P_i + \sum_{i \neq j} O_{ij} \rho_{ji} \tag{1.27}$$

Then we obtain two sums. One of them includes the contributions of the diagonal components of $\hat{O}$ and $\hat{\rho}$,

$$\Sigma^D = \sum_i O_i P_i \tag{1.28}$$

The other sum includes the contributions of the non-diagonal components of $\hat{O}$ and $\hat{\rho}$,

$$\Sigma^{ND} = \sum_{i \neq j} O_{ij} \rho_{ji} \tag{1.29}$$

It is not difficult to see that Σ^D has the structure of a classical expectation value. On the other hand, Σ^{ND} does not have this structure since, for example, ρ_{ji} may be a non-positive number. Thus, the term Σ^{ND} is an obstacle to interpret the quantum expectation value as a classical expectation value. That fact is reasonable because otherwise there would be nothing special about quantum mechanics. The sum Σ^{ND} embodies the most curious characteristic of quantum mechanics: the phenomenon of interference. For this reason, we call the terms of Σ^{ND} 'interference terms'. As a consequence, any attempt to find a limit between quantum statistics and classical statistics should include a process by which the interference terms disappear in the expectation values, i.e.,

$$\Sigma^{ND} \to 0 \tag{1.30}$$

We will call this process 'decoherence'. Thus it is possible build the following scheme:

$$\begin{array}{cl}
\left\langle \hat{O} \right\rangle_{\hat{\rho}} = \Sigma^D + \Sigma^{ND} & \sim \quad \text{quantum statistics} \\
\downarrow & \\
\text{decoherence} & \\
\downarrow & \\
\left\langle \hat{O} \right\rangle_{\hat{\rho}} = \Sigma^D & \\
\downarrow & \\
\text{interpretation} & \\
\left\langle \hat{O} \right\rangle_{\hat{\rho}} = \sum_i o_i P_i & \sim \quad \text{classical statistics}
\end{array} \tag{1.31}$$

At this point it is convenient to make an observation related with the orthodox approach to decoherence. The condition to obtain the limit between classical and quantum statistics is that, after a certain time, $\Sigma^{ND} = 0$. This is particularly true when the state operator is diagonal. In fact, when all the observables of the system are considered,

(i) if $\hat{\rho}$ is diagonal, then the interference terms of the expectation values of all the observables disappear, and

(ii) if the interference terms of the expectation values of all the observables disappear, then $\hat{\rho}$ is diagonal.

This means that, when we consider all observables, the study of decoherence can undertaken from two equivalent points of view:

a. By studying the diagonalization of the state.
b. By studying the vanishing of the interference terms from the expectation values.

5.1 Decoherence from the closed-system perspective

As emphasized by Omnés [35], decoherence is just a particular case of the general problem of irreversibility in quantum mechanics. The problem of irreversibility can be roughly expressed in the following terms. Since the quantum state $\hat{\rho}(t)$ follows an unitary evolution, it cannot reach a final equilibrium state for $t \to \infty$. Therefore, if the non-unitary evolution towards equilibrium is to be accounted for, a further element has to be added to the unitary evolution. From the most general viewpoint, this element consists in the splitting of the maximal information about the system into a relevant part and an irrelevant part: whereas the irrelevant part is disregarded, the relevant part is retained and its evolution may reach a final equilibrium situation.

This broadly expressed idea can be rephrased in operator's language. The maximal information about the system *U* is given by the space $\mathcal{O}$ of all its potentially possible observables. By selecting a particular subset $\mathcal{O}_R$ of this space, we restrict the maximal information to a relevant part: the expectation values $\langle \hat{O}_R \rangle_{\hat{\rho}}$ of the observables belonging to $\mathcal{O}_R \subset \mathcal{O}$ express the relevant information about the system. Of course, the decision about which observables are to be considered as relevant depends on the particular purposes in each situation; but without this restriction, irreversible evolutions cannot be described. A frequent choice of $\hat{O}_R \in \mathcal{O}_R$ is the one that splits the closed system *U*, represented in $\mathcal{H}$, into two open subsystems *S* and *E*, represented in $\mathcal{H}_S$ and $\mathcal{H}_E$ respectively, such that $U = S \cup E$. In this case:

$$\hat{O}_R = \hat{O}_S \otimes \hat{I}_E \tag{1.32}$$

where $\hat{I}_E$ is the identity of $\mathcal{H}_E \otimes \mathcal{H}_E$, and $\hat{O}_S \in \mathcal{H}_S \otimes \mathcal{H}_S$ is an observable of the system *S*. Since the identity $\hat{I}_E$ is the only observable of the subsystem *E* considered in this case, it is clear that

$\hat{O}_R$ only provides information about *S*. Therefore, the expectation values $\langle \hat{O}_R \rangle_{\hat{\rho}(t)}$ give only an account of the relevant part of the system. It is also essential to notice that, in principle, the decision about which are the relevant observables, i.e. those considered of interest, depends on the purpose of each situation.

On the basis of the concepts presented in the above paragraph, now we can describe the phenomenon of decoherence in general terms. The vanishing of the non-diagonal terms Σ^{ND} of the expectation value can be analyzed from the closed-system perspective by means of the expectation values. Given an initial state $\hat{\rho}(0)$ and the relevant observables belonging to $\mathcal{O}_R \subset \mathcal{O}$, decoherence takes place when $\Sigma^{ND}(t) \to 0$. This does not mean that the state operator becomes diagonal, but only that there are no interference terms for all the relevant observables. In other words, we can say that the initial state $\hat{\rho}(0)$ evolves as $\hat{\rho}(t)$ and, after a time t_D called 'decoherence time', the non-diagonal terms of the expectation values vanish.

This way of defining decoherence is different from the orthodox one, but includes it. In fact, when we select the relevant observables of the form $\hat{O}_R = \hat{O}_S \otimes \hat{I}_E$, then requiring that the interference terms disappear from the expectation value is equivalent to requiring that the reduced state operator of the system is diagonal:

(i) when $\hat{\rho}_S$ is diagonal, the interference terms disappear from the expectation values of all the relevant observables, and

(ii) if the interference terms disappear from the expectation values of all the relevant observables, then $\hat{\rho}_S$ is diagonal.

Nevertheless, when we select the relevant observables otherwise, the equivalence is lost. The closed-system approach leads to a more general framework for decoherence. In particular, it allows us to define the decoherence of groups of observables that do not have an associated state operator.

5.2 The General Theoretical Framework for Decoherence

In previous works we have analyzed the common characteristics of the different approaches of decoherence. The result of this analysis suggests the possibility of formulating a general framework for decoherence, such that those particular approaches can all be framed in it (see [29], [34], [36] and [37]). According to this general framework, which was developed in [36] and will be completed in future papers, decoherence is just a particular case of the general problem of irreversibility in quantum mechanics. Once the essential role played by the selection of the relevant observables is clearly understood, the phenomenon of decoherence can be explained in four general steps:

1. **First step**: The space $\mathcal{O}_R \subset \mathcal{O}$ of the relevant observables is defined.
2. **Second step**: The expectation value $\langle \hat{O}_R \rangle_{\hat{\rho}_R}$, for any $\hat{O}_R \in \mathcal{O}_R$, is obtained. This step can be formulated in two different but equivalent ways:
   a. A coarse-grained state $\hat{\rho}_R(t)$ is defined by
   $$\langle \hat{O}_R \rangle_{\hat{\rho}(t)} = \langle \hat{O}_R \rangle_{\hat{\rho}_R(t)} \tag{1.33}$$
   for any $\hat{O}_R \in \mathcal{O}_R$, and its non-unitary evolution governed by a master equation is computed (this step is typical in EID).
   b. $\langle \hat{O}_R \rangle_{\hat{\rho}(t)}$ is computed and studied as the expectation value of $\hat{O}_R$ in the state $\hat{\rho}(t)$. This is the generic case for other formalisms.
3. **Third step**: It is proved that $\langle \hat{O}_R \rangle_{\hat{\rho}(t)} = \langle \hat{O}_R \rangle_{\hat{\rho}_R(t)}$ reaches a final equilibrium value $\langle \hat{O}_R \rangle_{\hat{\rho}*}$. Then,
   $$\lim_{t\to\infty} \langle \hat{O}_R \rangle_{\hat{\rho}(t)} = \langle \hat{O}_R \rangle_{\hat{\rho}*} \qquad \forall \hat{O}_R \in \mathcal{O}_R \tag{1.34}$$
   This also means that the coarse-grained state $\hat{\rho}_R(t)$ evolves towards a final equilibrium state:
   $$\lim_{t\to\infty} \langle \hat{O}_R \rangle_{\hat{\rho}_R(t)} = \langle \hat{O}_R \rangle_{\hat{\rho}_R*} \qquad \forall \hat{O}_R \in \mathcal{O}_R \tag{1.35}$$
   The characteristic time for these limits is t_R, the relaxation time.
4. **Fourth step**: We decompose the expectation values into the classical part and the interference part, and we study whether the interference terms disappear. In this case, it is proved that

$$\left\langle \hat{O} \right\rangle_{\hat{\rho}} \to \sum_i O_i P_i \tag{1.36}$$

The characteristic time for this limit is the t_D, the decoherence time.

The limits of the third and the fourth steps mean that, although the off-diagonal terms of $\hat{\rho}(t)$ never vanish through the unitary evolution, the system decoheres from an observational point of view, that is, from the viewpoint given by any relevant observable $\hat{O}_R \in \mathcal{O}_R$.

From this general perspective, the phenomenon of destructive interference, which produces decoherence, is relative, because the off-diagonal terms of $\hat{\rho}(t)$ and $\hat{\rho}_R(t)$ vanish only from the viewpoint of the relevant observables $\hat{O}_R \in \mathcal{O}_R$. The essential difference between EID and other formalisms for decoherence is the selection of the relevant observables (see [36] for details). In fact, in EID approach the relevant observables are those that can be expressed as:

$$\hat{O}_R = \hat{O}_S \otimes \hat{I}_E \tag{1.37}$$

where the $\hat{O}_S$ are the observables of the system and $\hat{I}_E$ is the identity operator of the environment. Then, eq. (5.10) reads:

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = \left\langle \hat{O}_R \right\rangle_{\hat{\rho}_R(t)} = \left\langle \hat{O}_S \right\rangle_{\hat{\rho}_S(t)} \quad \text{where } \hat{\rho}_S(t) = Tr_E\left(\hat{\rho}(t)\right) \tag{1.38}$$

In the other formalisms, different restrictions in the set of observables are introduced.

5.3 The measurement problem from the closed system perspective

When the measurement process is studied from the point of view of the closed system, the system to be measured and the measuring apparatus form a complete quantum system, in which the identification of subsystems is not necessary. Instead, there are observables of the form $\hat{O}_S \otimes \hat{I}_M$ linked to the degrees of freedom of interest (the system of interest in EID). On the other hand, there is an observable $\hat{I}_S \otimes \hat{R}_M$ linked to the degrees of freedom of the pointer. As in Section 3.2, the interaction introduces a correlation between the eigenstates $|a_i\rangle$ of A and the eigenstates $|r_i\rangle$ of R:

$$|\psi_0\rangle = \sum_i c_i |a_i\rangle \otimes |r_0\rangle \quad \to \quad |\psi\rangle = \sum_i c_i |a_i\rangle \otimes |r_i\rangle \tag{1.39}$$

Now the problem can be reinterpreted from the new general framework for decoherence. After measurement, and considering the interaction with the environment, the complete whole system is in the superposition state

$$\hat{\rho}_{SME} = |\Psi_{SME}\rangle\langle\Psi_{SME}| = \sum_{i,j=1}^{N} c_i c_j^* |a_i\rangle \otimes |r_i\rangle \otimes |\varepsilon_i\rangle\langle\varepsilon_j| \otimes \langle r_j| \otimes \langle a_j| \tag{1.40}$$

However, if we select the relevant observable of the form

$$\hat{O}_R = \hat{I}_S \otimes \sum_{i=1}^{N} r_i |r_i\rangle\langle r_i| \otimes \hat{I}_E \tag{1.41}$$

the expectation value takes the following form

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}_{SME}} = \sum_{i=1}^{N} |c_i|^2 \, r_i \tag{1.42}$$

where the r_i are the possible values of pointer, $|c_i|^2 < 1$ and $\sum_{i=1}^{N} |c_i|^2 = 1$. Thus, we obtain a expectation value with a classical structure. If we only consider the expectation values of the relevant observable, there is no way that an experiment reveals a difference between the expectation values of the expression (5.19) and the expectation values that would be obtained if the state of the system were a true classical mixture. Therefore, the new approach of decoherence can explain why, if you perform a series of measurements, the expectation values adopt a classical structure. However, as the only state that we consider is the state of the whole closed system, this case is free from the conceptual difficulties of the EID approach.

5.4 The appearance of the classical world

As we have noticed above, the Correspondence Principle establishes that it should be possible to recover the laws of classical mechanics from the quantum laws. Historically, the aim was to try to transform a diagonal quantum state $\hat{\rho}$ into a state $\rho(p,q)$ of classical statistical mechanics. In this sense, the new approach for decoherence clearly shows that the quantum character of a system never vanishes: closed systems evolve always unitarily and, therefore, any kind of limit is explicitly impossible. In particular, the non-unitary evolution of the reduced state $\hat{\rho}_S$ is nothing else but a compact way of expressing the time evolution of the expectation values of the relevant observables.

However, the new approach is able to explain the appearance of the classical world as presented to the experience. When we select certain set of relevant observables $\mathcal{O}_R \subset \mathcal{O}$, and we verify that for them there is decoherence, then the quantum-classical transition occurs at the level of the expectation values $\langle \hat{O}_R \rangle_{\hat{\rho}}$, but not in the state. This means that the system behaves classically from an observational point of view. Classicality manifests itself or not, depending on the relevant observables under consideration. [31].

5.5 The generalized spin-bath model from the closed system perspective

Now we can analyze the spin-bath model from the closed-system perspective. The closed system $U = A \cup B$ is associated with a state $|\psi(t)\rangle$, which is the only legitimate quantum state.

In Decomposition 1, the EID approach conceives A as the open system S, and B as the environment E. In our framework, this case corresponds to selecting the relevant observables $\hat{O}_R$ of U of this form:

$$O_R = O_S \otimes I_E = O_A \otimes \left(\bigotimes_{i=1}^{N} I_i \right) \tag{1.43}$$

When the expectation values $\langle O_R \rangle_{\psi(t)} = \Sigma^d + \Sigma^{nd}(t)$ of the observables O_R in the state $|\psi(t)\rangle$ are computed (se eq.(4.4)), two cases can be distinguished (see [33] and [31]):

- In Case (a), $N \gg M$: Numerical simulations show that $\Sigma^{nd}(t) \to 0$ very fast for increasing time. This means that the observables given by (5.20) decohere when $N \gg M$.
- In Case (b), $M \gg N$ or $M \simeq N$: Numerical simulations show that $\Sigma^{nd}(t)$ exhibits an oscillating behavior. This means that the observables given by (5.20) do not decoheres when $M \gg N$ or $M \simeq N$.

In Decomposition 2 the EID approach conceives A_M as the open system S, and $\left(\bigcup_{i=1}^{M-1} A_i\right) \cup B$ as the environment E. In our framework, this case corresponds to selecting the relevant observables $\hat{O}_R$ of U of this form:

$$O_R = O_S \otimes I_E = O_{A_M} \otimes \left(\left(\bigotimes_{i=1}^{M-1} I_i \right) \otimes \left(\bigotimes_{k=1}^{N} I_k \right) \right) \tag{1.44}$$

When the expectation values $\langle O_R \rangle_{\psi(t)} = \Sigma^d + \Sigma^{nd}(t)$ of the observables O_R in the state $|\psi(t)\rangle$ are computed (see eq.(4.4)), numerical simulations show that, if $N \gg 1$, then $\Sigma^{nd}(t) \to 0$ very fast for increasing time (see [33] and [31]). This means that the observables given by (5.21) decohere when $N \gg 1$.

When we adopt this closed-system perspective, it turns out to be clear that there is no essential criterion for identifying the "open system" and its "environment". Some observable decoheres and others not. As the only legitimate quantum system is a closed system, the problem of splitting the whole system into system and environment, which challenges the EID approach, simply disappears.

6. Conclusions

In this paper we have studied the properties of open systems and their ontological status. We have recalled that the ontological interpretations of a proper and an improper mixture are very different: although $\hat{\rho}$ and $\hat{\rho}_R$ have similar mathematical structure, they do not refer to the same physical entity. Then, we considered how the reduced states of open systems are used in the phenomenon of decoherence in two different cases: the measurement process and those situations where the whole closed system can be split into an open system of interest and its environment in different ways. In the first case we have argued that, since the reduced state cannot be interpreted as a proper mixture, decoherence does not solve the measurement problem. In the second case we showed that the lack of a criterion to identify the open system and its environment is a manifestation of the relative nature of decoherence, and prevents us from conceiving the open system as a physical entity of the same ontological status as that of the closed system. Finally, we proposed a formalism designed to study the phenomena of relaxation and decoherence from a closed-system perspective. Since this formalism does not resort to reduced states, it avoids the interpretive problems that threaten the EID approach.

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